Nontempered Restriction Problems for Classical Groups

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January 26, 2021

Gross-Prasad Conjecture

The restriction problem referred to in the title is that arising in the Gross-Prasad conjecture.

Let $G_n = \mathrm{GL}_n$, U_n or SO_n over a local field F.

For $\pi \in \mathrm{Irr}(G_n)$ and $\sigma \in \mathrm{Irr}(G_{n-1})$, define a branching multiplicity

$$m(\pi, \sigma) = \dim \mathrm{Hom}_{G_{n-1}}(\pi, \sigma).$$

Have the following multiplicity-at-most-one result:

Theorem (Aizenbud-Gourevitch-Rallis-Schiffman (2010), Sun-Zhu (2012))

$$m(\pi, \sigma) \leq 1.$$

The GP conjecture proposes a determination of $m(\pi, \sigma)$ when π and σ belong to generic L-packets.

$\mathrm{GL}_n \times \mathrm{GL}_{n-1}$

Theorem

If π and σ are both generic (i.e. supports nonzero Whittaker functionals), then

$$m(\pi, \sigma) = 1.$$

In particular, the theorem covers every tempered irrep. of GL_n.

Question: What if $\pi \otimes \sigma$ is not generic?

In the following recent paper:

- W.T. Gan, B.H. Gross and D. Prasad, *Branching laws for classical groups: the nontempered case*, Compositio 156 (2020), 2298-2367.

We proposed an answer to this question for representations in the automorphic spectrum.

Automorphic Spectrum

These are unitary representations of $\mathrm{GL}_n(F)$ which occur in the spectral decomposition of the unitary representation

$$L^2_\chi(\mathrm{GL}_n(k)\backslash \mathrm{GL}_n(\mathbb{A}))$$

for some global field k with $F = k_v$ for some place v.

For $G = \mathrm{GL}_n$, denote the automorphic spectrum by

$$\widehat{G}^{temp} \subset \widehat{G}^{\mathrm{aut}} \subset \widehat{G}^{\mathrm{unit}} \quad \text{(Unitary dual).}$$

Arthur's conjecture give a classification of the irreducible summands of $L^2(G(k)\backslash G(\mathbb{A}))$, in terms of the notion of A-parameters.

Arthur Parameters

A local A-parameter for GL_n over a p-adic F is a conjugacy class of maps

$$\psi : \left(W_F \times \mathrm{SL}_2(\mathbb{C})^D\right) \times \mathrm{SL}_2(\mathbb{C})^A \longrightarrow \mathrm{GL}_n(\mathbb{C})$$

such that

$$\psi(W_F) \text{ is bounded in } \mathrm{GL}_n(\mathbb{C}).$$

Can write:

$$\psi = \bigoplus_i M_i \otimes [a_i] \otimes [b_i]$$

where M_i an irrep of W_F (bounded determinant) and $[a]$ the irrep. of SL_2 of dim. a, or

$$\psi = \bigoplus_{d \geq 1} M_d \otimes [d],$$

where M_d a rep. of $WD_F = W_F \times \mathrm{SL}_2^D$.

Type of A-parameters

The restriction of ψ to $\mathrm{SL}_2(\mathbb{C})^A$ corresponds to a unipotent conjugacy class in $\mathrm{GL}_n(\mathbb{C})$, i.e. a partition of n. We call this the type of ψ.

More precisely, if $\psi = \oplus_i M_d \otimes [d]$, then the type of ψ is the partition

$$(d^{\dim M_d})_{d \geq 1} = (1^{\dim M_1}, 2^{\dim M_2}, \ldots\ldots..).$$

For example: if ψ is trivial on SL_2^A, then ψ is simply a tempered L-parameter, and its type is trivial.

One can view the maximum dimension of irreps of SL_2^A occurring in ψ as a measure of nontemperedness of ψ.

L-parameter associated to an A-parameter

To each A-parameter ψ, we can associate an L-parameter

$$\phi_\psi : W_F \times \mathrm{SL}_2^D \longrightarrow W_F \times \mathrm{SL}_2^D \times \mathrm{SL}_2^A \longrightarrow \mathrm{GL}_n(\mathbb{C})$$

given by

$$\phi_\psi(w, g) = \psi\left(w, g, \begin{pmatrix} |w|^{1/2} & \\ & |w|^{-1/2} \end{pmatrix}\right).$$

The map $\psi \mapsto \phi_\psi$ gives an injection

$$\{\text{tempered } L\text{-parameters}\} \hookrightarrow \{A\text{-parameters}\} \hookrightarrow \{L\text{-parameters}\}$$

The L-parameters of the form ϕ_ψ are said to be L-parameters of Arthur type.

Arthur Packets

Arthur's conjecture postulates that to each ψ, one can associate a finite (multi-)set Π_ψ of irreducible unitary representations (in the automorphic discrete spectrum). One basic property that Π_ψ should have is

$$\Pi_\psi \supset \Pi^L_{\phi_\psi} \quad \text{(L-packet associated to } \phi_\psi\text{)}.$$

For GL_n, it turns out that

$$\Pi_\psi = \Pi^L_{\phi_\psi}$$

and in particular is a singleton. Hence

$$\widehat{\mathrm{GL}_n}^{\mathrm{aut}} = \{\Pi_\psi : \psi \text{ an } A\text{-parameter}\}.$$

$$= \bigsqcup_{\text{type } P} \widehat{\mathrm{GL}_n}^{\mathrm{aut}}_P.$$

The representations in Π_ψ are said to be of Arthur type and can be constructed as follows.

Representations of Arthur type

Suppose first that ψ is irreducible:

$$\psi = \rho \otimes [a] \otimes [b],$$

with ρ irrep. of W_F and $[a]$ the irrep. of SL_2 of dim. a. Let

$$\mathrm{St}(\rho, a) = \text{generalized Steinberg rep. with L-parameter } \rho \otimes [a]$$

$$= \text{unique irred. submodule of } \pi_\rho|-|^{(a-1)/2} \times \ldots \times \pi_\rho|-|^{-(a-1)/2}.$$

Then Π_ψ is the unique irred. quot. $\mathrm{Speh}(\rho, a, b)$ of

$$\mathrm{St}(\rho, a)|-|^{(b-1)/2} \times \ldots.. \times \mathrm{St}(\rho, a)|-|^{-(b-1)/2}.$$

If $\psi = \oplus_i \psi_i$ with ψ_i irreducible, then

$$\Pi_\psi = \Pi_{\psi_1} \times \ldots. \times \Pi_{\psi_r}.$$

Example: $\psi = [n]$ gives trivial rep. of GL_n.

Unitary Restriction Problem

Consider the direct integral decomposition of Π_ψ restricted to GL_{n-1}:

$$\Pi_\psi|_{\mathrm{GL}_{n-1}} = \int_{\widehat{\mathrm{GL}_n}^{\mathrm{unit}}} m(\sigma) \cdot \sigma \, d\mu_\psi(\sigma)$$

One is interested in understanding the support of the spectral measure $d\mu_\psi$:

$$\mathrm{supp}(d\mu_\psi) \subset \widehat{\mathrm{GL}_{n-1}}^{\mathrm{unit}}.$$

Theorem (Burger-Sarnak)

$$\mathrm{supp}(d\mu_\psi) \subset \widehat{GL_{n-1}}^{\mathrm{aut}}.$$

Theorem (Clozel)

All representations in $\mathrm{supp}(d\mu_\psi)$ are of the same type Q. Moreover, Q depends only on the type P of ψ.

Results of Venkatesh

Clozel's theorem implies there is map

$$f : \{\text{Partitions of } n\} \longrightarrow \{\text{Partitions of } n-1\}$$

such that any Arthur type representation of type P ion GL_n is supported on Arthur type representations of type $f(P)$ when restricted to GL_{n-1}.

Theorem (Venkatesh)

$$f(n_1, n_2, .., n_r) = (n_1 - 1, n_2 - 1, ..., n_r - 1, \quad 1, ...1)$$

Here we discard those $n_i - 1$ which are 0, and we add the appropriate number of 1's.

Indeed, Venkatesh considered a number of such problems: restriction from GL_n to GL_m, induction from GL_m to GL_n and tensor product of two reps. of GL_n.

Conjecture and Theorem

In our Compositio paper, Gross, Prasad and I made the following conjecture

Conjecture

Let ψ (resp. ψ') be an A -parameter of GL_n (resp. GL_{n-1}). Then

$$m(\Pi_\psi, \Pi_{\psi'}) = 1 \Longleftrightarrow (\psi, \psi') \text{ is a relevant pair of A-parameters}$$

We verified our conjecture when, for example, $\psi|_{\mathrm{SL}_2^D}$ and $\psi'|_{\mathrm{SL}_2^D}$ are trivial. Subsequently, Max Gurevich showed

$$m(\Pi_\psi, \Pi_{\psi'}) = 1 \Longrightarrow (\psi, \psi') \text{ is relevant },$$

and shortly after, Kei Yuen Chan showed

Theorem (KY Chan)

The above Conjecture holds.

Relevant A-parameters

Given A-parameters ψ and ψ' of GL_n and GL_{n-1}, we say (ψ, ψ') are relevant if we may write

$$\psi = \left(\bigoplus_{i \in I} \phi_i \otimes [b_i] \right) \oplus \left(\bigoplus_{j \in J} \rho_j \otimes [c_j - 1] \right)$$

and

$$\psi' = \left(\bigoplus_{i \in I} \phi_i \otimes [b_i - 1] \right) \oplus \left(\bigoplus_{j \in J} \rho_j \otimes [c_j] \right).$$

with $b_i, c_j \geq 1$.

Examples:

- when all b_i and $c_j = 1$: tempered/generic case
- when all $c_j = 1$: Venkatesh's theorem.

We may think of ψ and ψ' as "within distance 1 of each other".

Another formulation

Write

$$\psi = \bigoplus_d M_d \otimes [d] \quad \text{and} \quad \psi' = \bigoplus_d N_d \otimes [d].$$

Then (ψ, ψ') is relevant if there are decompositions of WD_F-modules:

$$M_d = M_d^+ \oplus M_d^- \quad \text{and} \quad N_d = N_d^+ \oplus N_d^-$$

such that

$$M_d^+ = N_{d+1}^- \quad \text{and} \quad M_d^- = N_{d-1}^+.$$

This formulation is more convenient for extending the notion of relevance to classical groups.

Examples

(1) When $\psi = [n]$, Π_ψ is the trivial rep. of GL_n, so we know its restriction to GL_{n-1}. The only ψ' such that (ψ, ψ') is relevant is

$$\psi' = [n-1].$$

(2) Now take $\psi' = [n-1]$, so that $\Pi_{\psi'}$ is the trivial rep. of GL_{n-1}. The only ψ's for which (ψ, ψ') is relevant are:

- $\psi = [n]$
- $\psi = [n-2] + \rho$, with 2-dim tempered ρ.

Relevance and Correlator

Zhiwei Yun has given a more conceptual formulation of the notion of relevance. Given

$$\psi : WD_F \times \mathrm{SL}_2^A \longrightarrow \mathrm{GL}(V) \quad \text{and} \quad \psi' : WD_F \times \mathrm{SL}_2^A \longrightarrow \mathrm{GL}(W),$$

the maximal torus of SL_2^A gives a $\mathbb{C}^\times$-action and hence a WD_F-stable $\mathbb{Z}$-grading on V and W. Then $\mathrm{End}(V)$, $\mathrm{End}(W)$, $\mathrm{Hom}(V,W)$ and $\mathrm{Hom}(W,V)$ also inherit a grading.

The unipotent elements

$$e = \psi\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \in \mathrm{End}(V) \quad \text{and} \quad e' = \psi'\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \in \mathrm{End}(W).$$

are elements of degree 2.

Lemma

The pair (ψ, ψ') is relevant if and only if there exists degree 1 elements $T \in \mathrm{Hom}_{WD_F}(V,W)$ and $S \in \mathrm{Hom}_{WD_F}(W,V)$ such that

$$e = S \circ T \quad \textit{and} \quad e' = T \circ S.$$

Relevance and Moment Map

One can reformulate Yun's interpretation in more geometric terms, in terms of a moment map arising in theta correspondence. The group $\mathrm{GL}(V) \times \mathrm{GL}(W))$ acts on the symplectic variety

$$T^*(V^* \otimes W) = (V^* \otimes W) \times (V \otimes W^*).$$

This action is Hamiltonian and thus give rise to moment maps:

$$\begin{tikzcd}[column sep=small] & \mathrm{Hom}(V, W) \times \mathrm{Hom}(W, V) \arrow[dl, "p"'] \arrow[dr, "q"] & \\ \mathfrak{gl}(V) & & \mathfrak{gl}(W) \end{tikzcd}$$

where

$$p(T, S) = S \circ T \quad \text{and} \quad q(T, S) = T \circ S.$$

Hence (ψ, ψ') is relevant iff there exists a WD_F-invariant degree 1 element (T, S) such that $p(T, S) = e$ and $q(T, S) = e'$.

Philosophy of Sakellaridis-Venkatesh

We can view the previous discussion through the philosophy of Sakellaridis-Venkatesh in the relative Langlands program.

Problem: If $X = H\backslash G$ is a G-spherical variety, describe

- the decomposition of $L^2(X)$;
- the subset $\mathrm{Irr}_X(G) = \{\pi \in \mathrm{Irr}(G) : \pi \text{ is } H\text{-distinguished}\}$.

[SV] formulates a conjectural answer:

- there is a dual group $X^\vee$ equipped with $\iota : X^\vee \times \mathrm{SL}_2 \to G^\vee$;
- there is a $\mathbb{Z}$-graded finite-dim. rep. V_X of $X^\vee$,

so that

- representations in $L^2(X)$ or $\mathrm{Irr}_X(G)$ are those whose A-parameters factor through ι;
- the spectral measure of $L^2(X)$ is described by the L-function associated to V_X.

In our case, $X = (GL_n \times \mathrm{GL}_{n-1})/\mathrm{GL}_{n-1}^{\Delta}$ and $X^\vee = G^\vee = \mathrm{GL}_n \times \mathrm{GL}_{n-1}$

Duality of Hamiltonian G-varieties

Recently, [SV] began a reformulation as a duality between certain Hamiltonian G-varieties.

- Starting from a Hamiltonian G-variety Y, one can attach a dual Hamiltonian $G^\vee$-variety $Y^\vee$;
- A Hamiltonian G-variety can be quantized to a unitary rep. Π_Y of G;
- the spectral decomposition of Π_Y, or the description of

$$\mathrm{Irr}_Y(G) = \{\pi \in \mathrm{Irr}(G) : \mathrm{Hom}_G(\Pi_Y^\infty, \pi) \neq 0\},$$

  is governed by the symplectic geometry of $Y^\vee$.

When X is a G-spherical variety, let $Y = T^*(X)$. Then the quantization of Y is

$$\Pi_Y = L^2(X) \quad \text{and} \quad \Pi_Y^\infty = \mathcal{S}(X).$$

Then $Y^\vee$ is something like

$$Y^\vee = G^\vee \times_{X^\vee} V_X$$

at least when $\iota(\mathrm{SL}_2)$ is trivial.

Our Example

In our case, $X = (GL_n \times \mathrm{GL}_{n-1})/\mathrm{GL}_{n-1}^{\Delta}$,

$$X^{\vee} = G^{\vee} = \mathrm{GL}_n \times \mathrm{GL}_{n-1} \quad \text{and} \quad V_X = V^* \otimes W \times V \otimes W^*.$$

So

$$Y = T^*(X) \quad \text{and} \quad Y^{\vee} = V^* \otimes W \times V \otimes W^* = T^*(V^* \otimes W).$$

Our conjecture gives a precise formulation of the expectation:

The symplectic geometry of $Y^{\vee}$ (via its moment map) governs the decomposition of the quantization of Y.

We may exchange the role of Y and $Y^{\vee}$. The quantization of $Y^{\vee}$ is

$$L^2(V^* \otimes W) = \text{the Weil rep. of } \mathrm{GL}(V) \times \mathrm{GL}(W).$$

So we expect the theta correspondence to be governed by the symplectic geometry of Y! Hence,

Gross-Prasad periods is dual to theta correspondence.

Classical Groups

For the rest of the talk, we consider the case of

$$G \times H = \mathrm{SO}_{2n+1} \times \mathrm{SO}_{2n}.$$

So

$$G^\vee = \mathrm{Sp}_{2n}(\mathbb{C}) \quad \text{and} \quad H^\vee = \mathrm{SO}_{2n}(\mathbb{C}).$$

Hence an A-parameter for G is a symplectic rep.

$$\psi : WD_F \times \mathrm{SL}_2^A \longrightarrow \mathrm{Sp}_{2n}(\mathbb{C})$$

whereas an A-parameter for H is an orthogonal rep.

$$\psi' : WD_F \times \mathrm{SL}_2^A \longrightarrow \mathrm{SO}_{2n}(\mathbb{C}).$$

We may write

$$\psi = \bigoplus_d M_d \otimes [d] \quad \text{and} \quad \psi' = \bigoplus_d N_d \otimes [d].$$

As before,(ψ, ψ') is defined to be relevant if ....

Relevance and Moment Map

With $G^\vee = \mathrm{Sp}(W)$ and $H^\vee = \mathrm{SO}(V)$, $G^\vee \times H^\vee$ acts naturally on the symplectic variety $W^* \otimes V$, giving the moment map diagram;

$$\begin{tikzcd} & W^* \otimes V \arrow[dl, "p"'] \arrow[dr, "q"] & \\ e \in \mathfrak{sp}(W) & & e' \in \mathfrak{so}(V) \end{tikzcd}$$

The pair of A-parameters (ψ, ψ') is relevant if and only if there exists a degree 1 element $T \in \mathrm{Hom}_{WD_F}(W, V)$ such that

$$T^* \circ T = e \quad \text{and} \quad T \circ T^* = e'$$

where $T^* \in \mathrm{Hom}_{WD_F}(V^*, W^*) \cong \mathrm{Hom}_{WD_F}(V, W)$ is the adjoint map.

A-Packets

Given A-parameter ψ of $G = \mathrm{SO}_{2n+1}$, Arthur associates an A-packet Π_ψ of finitely many unitary reps in the automorphic spectrum. But one knows much less about Π_ψ compared to the case of GL_n.

- If ϕ_ψ is the L-parameter associated to ψ, then $\Pi^L_{\phi_\psi} \subset \Pi_\psi$.
- Let

$$A_\psi = \pi_0(Z_{G^\vee}(\psi)).$$

Then Π_ψ is a representation of $G \times A_\psi$:

$$\Pi_\psi = \bigoplus_{\eta \in \mathrm{Irr}(A_\psi)} \eta \otimes \pi_\eta$$

where π_η is a unitary rep. of G of finite length (maybe 0). Moeglin has shown that Π_ψ is multiplicity-free in the p-adic case.

Complications

The complications for classical groups include

- Π_ψ is not a singleton, in general.
- while
$$\widehat{G}^{\mathrm{aut}} = \bigcup_{\psi} \Pi_\psi,$$
these unions are not disjoint. So a representation $\pi \in \widehat{G}^{\mathrm{aut}}$ may have multiple types.

For example, a nongeneric supercuspidal rep. belongs to a tempered L-packet but may also belong to a nontempered A-packet, such as a Saito-Kurokawa A-packet for SO_5.

However, a result of Moeglin says that an unramified rep. in $\widehat{G}^{\mathrm{aut}}$ has a well-defined type.

Questions

Given $\pi \in \mathrm{Irr}(G)$ and $\sigma \in \mathrm{Irr}(H)$ of Arthur type, we would like to determine

$$m(\pi, \sigma) = \dim \mathrm{Hom}_H(\pi, \sigma).$$

We may consider this at various levels:

- For which A-parameters (ψ, ψ') is

$$m(\psi, \psi') = \dim \mathrm{Hom}_H(\Pi_\psi, \Pi_{\psi'}) \neq 0?$$

- If $m(\psi, \psi')$ is nonzero, for which (η, η') is

$$m(\pi_\eta, \sigma_{\eta'}) \neq 0?$$

- Determine $m(\pi_\eta, \sigma_{\eta'})$ precisely.

Given the results in the GL setting, one might expect the following implication:

$$m(\psi, \psi') \neq 0 \Longrightarrow (\psi, \psi') \text{ is relevant.}$$

It turns out that this is not true.

A Counterexample

We will consider A-parameters ψ and ψ' which are trivial on W_F, so they are just reps. of $\mathrm{SL}_2^D \times \mathrm{SL}_2^A$. For any subset $J \subset \{1, 2, 3, ..., n\}$, set

$$\psi_J = \bigoplus_{i \notin J} [2i] \otimes [1] \oplus \bigoplus_{j \in J} [1] \otimes [2j].$$

This is an A-parameter for some SO_{2N+1}. We will take

$$\psi' = ([1] + [3] + ... + [2n-1]) \otimes [1].$$

so that ψ' is a discrete series L-parameter for some SO_{2M}.

We shall show that there is a supercuspidal representation

$$\pi \in \bigcap_J \Pi_{\psi_J}, \quad \text{such that} \quad m(\pi, \psi') = 1.$$

Hence, $m(\psi_J . \psi') \neq 0$ for any J, but

$$(\psi_J, \psi') \text{ is relevant} \iff J = \emptyset \text{ or } \{1\}.$$

Counterexample Continued

How to show the existence of π?
Let $\Delta : \mathrm{SL}_2^D \to \mathrm{SL}_2^D \times \mathrm{SL}_2^A$ be diagonal map and set $\psi_J^\Delta = \psi_J \circ \Delta$.
Observe:

$$\psi_J^\Delta = \rho := \bigoplus_{j=1}^{n} [2j] \quad \text{(a discrete series L-parameter indept. of } J\text{)}$$

We now use two results of Moeglin:

- The L-packet of Ψ_ρ has a unqiue supercuspidal member π.
- π lies in any A-packet Π_ψ for which $\psi^\Delta = \rho$.

This gives the desired supercuspidal $\pi \in \bigcap_J \Pi_{\psi_J}$.

To show $m(\pi, \psi') = 1$, apply the GP conjecture (proved by Waldspurger) to the tempered parameters (ρ, ψ').

Results of A. Hendrickson

A. Hendrickson has considered the L^2-restriction problem in his thesis work, extending the work of Venkatesh. Because unramified representations have well-defined types, Clozel's result gives a map

$$f : \{\text{Types for } \mathrm{SO}_{2n+1}\} \longrightarrow \{\text{Types for } \mathrm{SO}_{2n}\}$$

such that for any unramified rep. of SO_{2n+1} of type P, its spectral measure as a rep. of SO_{2n} is supported on reps. of type $f(P)$.

- a type for SO_{2n+1} is a partition of $2n$ in which every odd number appears even number of times.
- a type for SO_{2n} is a partition of $2n$ in which every even number appears even number of times.

Theorem

The map f is given by:

$$f(n_1, n_2, .., n_r) = (n_1 - 1, ..., n_r - 1, 1, 1..., 1)$$

The Conjecture for L-packets of Arthur Type

Given A-parameter (ψ, ψ'), consider its associated L-packet $\Pi^L_{\phi_\psi} \times \Pi^L_{\phi_{\psi'}}$. Then

$$m(\phi_\psi, \phi_{\psi'}) \neq 0 \iff (\psi, \psi') \text{ is relevant,}$$

in which case

$$m(\phi_\psi, \phi_{\psi'}) = 1$$

Moreover, the unique rep. in the L-packet with nonzero contribution is given as a character of the component group by the same recipe as in the generic case.

Thus, this conjecture may be viewed as the natural extension of GP from the class of tempered L-packets to the larger class of L-packets of Arthur type.

The Conjecture for A-packets

We believe:

- If ψ and ψ' are both trivial on SL_2^D, then

$$m(\psi, \psi') \neq 0 \Longleftrightarrow (\psi, \psi') \text{ is relevant.}$$

- If π and σ are reps of Arthur type, then $m(\pi, \sigma) \neq 0$ implies

$$\text{there exists relevant } (\psi, \psi') \text{ such that } \pi \in \Pi_\psi \text{ and } \sigma \in \Pi_{\psi'}$$

Observe that this conjecture is still far from answering the list of questions we listed earlier.

Example: Automorphic Descent

Let us explain how the theory of automorphic descent (by Ginzburg-Rallis-Soudry) and its extension (twisted automorphic descent by Jiang-Zhang) is explained by our conjectures.

Start with an elliptic global L-parameter for, say SO_{2n+1}:

$$\Pi = \oplus_i \Pi_i \quad \text{with } L(1, \Pi_i, \wedge^2) = \infty.$$

Goal: produce the generic cuspidal representation in this L-packet, and more generally the whole L-packet.

May think of Π as a cuspidal representation of $\prod_i \mathrm{GL}_{n_i}$, which is a Levi factor M of a parabolic subgroup of SO_{4n}.

Automorphic Descent

Construction:

- consider iterated residue at $(1/2, ..., 1/2)$ of the Eisenstein series of SO_{4n} associated to the induced rep Π on M. The residue is an square-integrable automorphic form $\mathrm{Res}(\Pi)$ with A-parameters

$$\Pi \otimes [2] = \bigoplus_i \Pi_i \otimes [2].$$

- Consider Bessel descent to SO_{2n+1}: for which elliptic A-parameter Σ of SO_{2n+1} is

$$\mathrm{Bessel}(\mathrm{Res}(\Pi), \Sigma) \neq 0?$$

The global analog of our conjecture says that the A-parameters of $\mathrm{Res}(\Pi)$ and Σ are relevant.

The only possibility is $\Sigma = \Pi$!.

THANK YOU FOR YOUR ATTENTION!