p-adic cohomology and arithmetic geometry 2018

Abstracts

Tomoyuki Abe: Nearby cycles for arithmetic $\mathscr{D}$-modules

I will define a nearby cycle functor for arithmetic $\mathscr{D}$-modules, and prove some basic properties. As an application, I will show that the $\mathscr{D}$-module theoretic pushforward of an isocrystal along a proper smooth morphism is an isocrystal.

Emiliano Ambrosi: Monodromy groups of convergent isocrystals and applications

I will report on a work in progress with Marco D'Addezio.
Let X_0 be a smooth, geometrically connected variety defined over a finite field $\mathbb{F}_q$ and M_0 an irreducible F-overconvergent on X_0 with constant Newton polygon. We show that if the sub object of minimal slopes admit a non trivial morphism to the constant isocrystals on X_0 (not necessarily preserving the Frobenius structure), then M_0 is isomorphic to a twist of the constant isocrystal. The key input in the proof is the study of the monodromy groups of (over)convergent isocrystals. They are defined via the Tannakian formalism and allow us to attack the problem via group-theoretic techniques. Our main result in this setting is that the reductive rank of the monodromy group of an overconvergent isocrystal is the same as the monodromy group of the associated convergent isocrystal.
As an application, we prove that an abelian variety defined over the function field $K(X)$ of X (here X is the base change of X_0 to an algebraic closure of $\mathbb{F}_q$), without isotrivial geometric isogeny factors, has only finitely many torsion points on the perfect closure of $K(X)$, giving a positive answer to a question of Esnault.

Oliver Gregory: Overconvergent Hyodo-Kato cohomology for semistable varieties

I shall explain an overconvergent refinement of the Hyodo-Kato complex whose hypercohomology computes log-rigid cohomology of semistable varieties. This is joint work with Andreas Langer.

Michael Groechenig: A higher-dimensional generalisation of the epsilon connection

De Rham epsilon lines for holonomic $\mathscr{D}$-modules on curves were introduced by Deligne and Beilinson-Bloch-Esnault. This formalism includes a product formula, expressing the determinant of cohomology of a holonomic $\mathscr{D}$-module as a tensor product of the epsilon lines computed with respect to a non-zero rational 1-form. Patel generalised the theory of de Rham epsilon factors to arbitrary dimensions. A curious feature of BBE 's 1-dimensional theory, is the epsilon connection which appears when studying the variation of the epsilon lines on the space of non-zero 1-forms. In this talk I will explain how properties of algebraic K-theory yield a conjectural candidate for the epsilon connection in arbitrary dimensions.

Shin Hattori: Duality of Drinfeld modules and P-adic properties of Drinfeld modular forms

Let p be a rational prime, $q > 1$ a p-power and P a non-constant irreducible polynomial in $\mathbb{F}_q[t]$. The notion of Drinfeld modular form is an analogue over $\mathbb{F}_q(t)$ of that of elliptic modular form.

On the other hand, following the analogy with p-adic elliptic modular forms, Vincent defined P-adic Drinfeld modular forms as the P-adic limits of Fourier expansions of Drinfeld modular forms. Numerical computations suggest that Drinfeld modular forms should enjoy deep P-adic structures comparable to the elliptic analogue, while at present their P-adic properties are far less well understood than the p-adic elliptic case. In this talk, I will explain how basic properties of P-adic Drinfeld modular forms are obtained from the duality theories of Taguchi for Drinfeld modules and finite v-modules.

Kazuhiro Ito: CM liftings of K3 surfaces over finite fields and the Tate conjecture

Using integral canonical models of orthogonal Shimura varieties and the Kuga-Satake construction, we will prove that any K3 surface of finite height over a finite field admits a characteristic 0 lifting whose generic fiber is a K3 surface with complex multiplication (CM). Combining our methods with the results of Mukai and Buskin on the Hodge conjecture for products of K3 surfaces, we will prove that the Tate conjecture for the square of a K3 surface over a finite field is true. This is a joint work with Tetsushi Ito and Teruhisa Koshikawa.

Wataru Kai: Algebraic cycles with modulus and some relative K-groups

Part of this talk is based on joint work with R. Iwasa and with H. Miyazaki. I will discuss parallel aspects of Binda-Saito's algebraic cycle theory and the K-theory of a smooth variety relative to an effective Cartier divisor. This includes the existence of a reasonable theory of Chern classes. As the divisor gets thicker and thicker, both groups tend to have large p-primary torsion subgroups (in positive characteristic p). In some cases, they are related to the de Rham-Witt complex.

Teruhisa Koshikawa: $\mathbf{A}_{\mathrm{inf}}$-cohomology in the semistable case

I will discuss the $\mathbf{A}_{\mathrm{inf}}$-cohomology in the case of semistable reduction, extending the work of Bhatt-Morrow-Scholze. Our cohomology theory interpolates the log crystalline cohomology, the log de Rham cohomology, and the etale cohomology. This is joint work with Kestutis Cesnavicius.

Joe Kramer-Miller: Ramification of p-adic etale sheaves coming from ordinary F-isocrystals on curves

Wan conjectured that the variation of zeta functions along towers of curves associated to the p-adic etale cohomology of a fibration of smooth proper ordinary varieties should satisfy several stabilizing properties. The most basic of these conjectures state that the genera of the curves in these towers grow in a regular way. We state and prove a generalization of this conjecture, which applies to the graded pieces of the slope filtration of an overconvergent F-isocrystal.

Christopher Lazda: Local acyclicity in p-adic cohomology

We prove an analogue for p-adic coefficients of the Deligne-Laumon theorem on local acyclicity for curves. That is, for an overconvergent F-isocrystal E on a relative curve $f\colon U \to S$ admitting a good compactification, we show that the cohomology sheaves of $Rf_!E$ are overconvergent isocrystals if and only if E has constant Swan conductor at infinity.

Isabel Leal: Refined Swan conductors mod p of one-dimensional Galois representations

In this talk, I will discuss refined Swan conductors mod p of one-dimensional Galois representations, which were introduced in a joint work with Kazuya Kato and Takeshi Saito. Time permitting, I will also go over some applications and related subjects.

Yuya Matsumoto: μ_p and α_p-actions on K3 surfaces in characteristic p

Actions of (finite or infinite) groups on K3 surfaces have been vastly studied. We consider a counterpart: actions of non-reduced group schemes, especially μ_p and α_p, on K3 surfaces in characteristic $p > 0$. In this talk we focus on actions whose quotients are again (birational to) K3 surfaces, and determine possible characteristics and singularities. It turns out that the behaviors of μ_p and α_p-actions are similar to those of $\mathbb{Z}/l\mathbb{Z}$ and $\mathbb{Z}/p\mathbb{Z}$-actions respectively.

Kazuaki Miyatani: p-adic hypergeometric $\mathscr{D}$-modules and multiplicative convolution

In this talk, we prove that the arithmetic $\mathscr{D}$-modules defined by p-adic hypergeometric differential equations can be described as an iterated multiplicative convolution of arithmetic $\mathscr{D}$-modules of rank one, under a "p-adic non-Liouvilleness" condition on the parameters. This is a p-adic counterpart of a result of N. M. Katz in the complex analytic case. Moreover, in the case where the parameters are rational, we use this result to prove that the p-adic hypergeometric differential equations define overconvergent F-isocrystals and that they are crystalline companions of l-adic hypergeometric sheaves constructed by Katz.

Kentaro Nakamura: Construction of zeta element for rank two universal deformation

In his celebrated work on Iwasawa main conjecture for modular forms, Kazuya Kato constructed an element in the Iwasawa cohomology of the p-adic Galois representations associated to Hecke eigen cusp new forms, and proved that this element is related to critical values of L-functions via dual exponential maps. Such an element is called zeta element, and it is predicted to exist even for families of p-adic Galois representations. For example, Ochiai, and recently Fukaya-Kato constructed zeta elements for Hida families. In this talk, we construct a zeta element for rank two universal deformations. Based on Fukaya-Kato's construction via modular symbols, we construct it using many deep results of p-adic (global and local) Langlands correspondence. In particular, we use Emerton's work on completed cohomology of modular curves and Paskunas' works on Colmez's functor and Breuil-Mézard conjecture.

Shun Ohkubo: Logarithmic growth filtrations for (φ, ∇)-modules over the bounded Robba ring

In this talk, we study the logarithmic growth (log-growth) filtration, a mysterious invariant found by B. Dwork, for (φ, ∇)-modules over the bounded Robba ring. The main result is a proof of a conjecture proposed by B. Chiarellotto and N. Tsuzuki on a comparison between the log-growth filtration and Frobenius slope filtration. One of the ingredients of the proof is a new criterion for pure of bounded quotient, which is a notion introduced by Chiarellotto and Tsuzuki to formulate their conjecture. We also give several applications to log-growth Newton polygons, including a conjecture of Dwork on the semicontinuity, and an analogue of a theorem due to V. Drinfeld and K. Kedlaya on Frobenius Newton polygons for indecomposable convergent F-isocrystals.

Daxin Xu: On higher direct images of convergent isocrystals

Let k be a perfect field of characteristic $p > 0$ and W the ring of Witt vectors of k. In this talk, we give a new proof of the Frobenius descent for convergent isocrystals on a variety over k relative to W. This proof allows us to deduce an analogue of the de Rham complexes comparison theorem of Berthelot without assuming a lifting of the Frobenius morphism. As an application, we prove a version of Berthelot's conjecture on the preservation of convergent isocrystals under the higher direct image by a smooth proper morphism of k-varieties in the context of Ogus' convergent topos.

Fuetaro Yobuko: On quasi-Frobenius split height and quasi-Frobenius split schemes

The quasi-Frobenius-split height of a scheme of positive characteristic is a quantification of the notion of Frobenius-splitting. For a Calabi-Yau variety, it is equal to the Artin-Mazur height, which is related to the part with Frobenius slopes less than one of the crystalline cohomology. In this talk, we will show some properties of quasi-Frobenius-split schemes (= a scheme with finite Frobenius-split height) and computations of the Frobenius-split height of some schemes except Calabi-Yau varieties.

Yasuhiro Wakabayashi: Symplectic aspects of the p-adic Teichmüller uniformization

The p-adic Teichmüller theory established by S. Mochizuki describes the uniformization of p-adic hyperbolic curves and their moduli. It may be regarded as an analogue of the Serre-Tate theory of ordinary abelian varieties. The essential ingredients for constructing canonical p-adic liftings of hyperbolic curves are certain flat projective bundles called ordinary nilpotent indigenous bundles. In this talk, we would like to give a quick review of a part of the p-adic Teichmüller theory. Also, we would like to explain symplectic aspects of the moduli space of ordinary nilpotent indigenous bundles, including a p-adic version of a comparison theorem of S. Kawai and B. Loustau.