p-adic cohomology and arithmetic geometry 2022

Abstracts

Bruno Chiarellotto: Multivariable de Rham representations, Sen theory and p-adic differential equations

(joint work with O. Brinon and N. Mazzari) Let K be a complete valued field extension of $\mathbb{Q}_p$ with perfect residue field. We consider p-adic representations of a finite product $G^{\Delta_K}; = G_K^{\Delta}$ of the absolute Galois group G_K of K. This product appears as the fundamental group of a product of diamonds. We develop the corresponding p-adic Hodge theory by constructing analogues of the classical period rings $\mathbb{B}_{\mathrm{dR}}$ and $\mathbb{B}_{\mathrm{HT}}$, and multivariable Sen theory. In particular, we associate to any p-adic representation V of G_K^{Δ}; an integrable p-adic differential system in several variables $\mathbb{D}_{\mathrm{dif}}(V)$. We prove that this system is trivial if and only if the representation V is de Rham. Finally, we relate this differential system to the multivariable overconvergent (φ, Γ)-module of V constructed by Pal and Zabradi along classical Berger's construction.

Marco D'Addezio: Crew's parabolicity conjecture

I will report on recent developments of the theory of monodromy groups of convergent and overconvergent F-isocrystals. These algebraic groups have been defined by Crew in '92 using the Tannakian formalism. After quickly recalling their construction I will explain the main ideas of the proof of Crew's parabolicity conjecture, which provides a strong relation between these two types of groups. I will continue by presenting some applications of the conjecture and I will finish stating a local enhancement I recently obtained in a joint work with Van Hoften.

Kiran S. Kedlaya: On the construction of crystalline companions (3 lectures)

We give some of the key details of the construction of crystalline companions of lisse l-adic étale sheaves on a smooth scheme over a finite field. In the three lectures, we focus on the following aspects.

1. Overview of the proof. This includes reduction steps coming from the existence of étale companions, a description of the key geometric setup using stable curve fibrations, and the principle of "tame rigidity": roughly, if a tame coefficient object on a single curve in a smooth fibration spreads out étale-locally on the base, then this spreading is unique. We also discuss the obstruction to spreading out in the étale case in terms of the exact sequence of fundamental groups associated to a fibration.

2. Moduli stacks of tame truncated crystals on a curve fibration and their relationship with tame overconvergent isocrystals. In addition to some careful work with Witt vectors, this requires observing Ogus's distinction between convergent and p-adically convergent isocrystals (an the fact that the difference disappears in the presence of a Frobenius structure).

3. Construction of crystalline companions on curve fibrations using moduli stacks of tame truncated crystals, focusing on the case where the base is itself a curve. In the isoclinic

case, we directly adapt Drinfeld's construction of étale companions. Otherwise, we use the minimal slope theorem of Tsuzuki-D'Addezio to reconstruct the étale obstruction in an abelian form, then eliminate the obstruction using Deligne's finiteness theorem for étale local systems on a curve.

Teruhisa Koshikawa: Logarithmic prismatic cohomology (lecture series)
Zijian Yao

In these two talks, we will discuss a logarithmic generalization of prismatic cohomology developed by Bhatt and Scholze. In particular, we will discuss how it specializes and compares to other cohomology theories (log de Rham, log crystalline and (Kummer) log etale cohomology). We will also discuss the notion of Nygaard filtration on it, along with some applications. These talks are based the earlier work Logarithmic Prismatic Cohomology I due to one of us (T. K.) and our recent joint work Logarithmic Prismatic Cohomology II.

Yuya Matsumoto: Inseparable analogue of Kummer K3 surfaces

A Kummer surface is the minimal resolution of the quotient of an abelian surface A by $[-1]$. This construction gives a K3 surface except precisely if A is supersingular of characteristic 2. It follows that there are no supersingular K3 surfaces in characteristic 2 that are Kummer surfaces. In this talk we will discuss possible analogues of the notion of Kummer surfaces, in terms of inseparable quotient morphisms by actions of μ_p and α_p, that can be applied to supersingular K3 surfaces in characteristic 2.

Kazuaki Miyatani: Cancellation theorem for p-adic hypergeometric sheaves

In this talk I will discuss arithmetic hypergeometric $\mathscr{D}$-modules with parameters possibly with integer differences (but without p-adic Liouville differences). Inspired by a study for complex and ℓ-adic cases by N. M. Katz, I will explain how such arithmetic $\mathscr{D}$-modules appear as an extension of some other hypergeometric objects. In the case where the parameters are rational, this fact also gives an interpretation of the cancellation theorem for hypergeometric functions over finite fields in terms of p-adic hypergeometric modules.

Yukiyoshi Nakkajima: Theory of Hirsch weight-filtered log crystalline complex

You can see the following theory on the arxiv. In this talk we give a theory of the derived PD-Hirsch extension of the log crystalline complex of a log smooth scheme and we construct a fundamental filtered complex (Hzar, P) for a simple normal crossing log scheme X over a family S of log points by using the log crystalline method in order to overcome obstacles arising from the incompatibility of the p-adic Steenbrink complex defined in my previously preprint with the cup product of the log crystalline complex of X. (In fact we can construct a fundamental filtered dga (Hzar,TW, P).) When the base log scheme is the log point of a perfect field of characteristic $p > 0$, we prove that (Hzar,TW, P) and (Hzar, P) is canonically isomorphic to Kim and Hain's filtered dga and their filtered complex in [KH], respectively. As applications of this theory, we overcome the obstacles above and we show fundamental properties of the weight filtration on the log crystalline cohomology sheaf of X when the underlying scheme of X is projective over the underlying scheme of S. Especially we prove the existence of a nondegenerate perfect paring for the log isocrystalline cohomology sheaf which is compatible with the weight filtration. We also prove that the p-adic variational filtered log hard Lefschetz conjecture with respect to the weight filtration is true when there exists a log fiber which is the log special fiber of a projective strict semistable family over a complete discrete valuation ring of any characteristic.

Noriyuki Otsubo: Hypergeometric motives and p-adic periods

We define Chow motives whose complex periods (resp. Frobenius traces) give generalized hypergeometric functions over the complex number field (resp. over finite fields). As the p-adic periods of these motives, we define B_{dR}-valued p-adic generalized hypergeometric functions.

Atsushi Shiho: Chern classes of (non locally free) crystals

We prove that, if a coherent sheaf on a projective smooth variety over a perfect field of positive characteristic comes from a p-torsion free crystal of finite presentation (which is not necessarily locally free), its rational crystalline Chern classes of positive degree all vanish. This result is already announced by Bhatt-Lurie. We give an alternative proof which is based on rigid cohomology.

Akio Tamagawa: Tannakian Chebotarev density theorems for overconvergent and convergent F-isocrystals

This is a joint work with Anna Cadoret. In this talk, we first prove the following Tannakian Chebotarev density theorem for overconvergent F-isocrystals. Let X be a normal variety of positive dimension over a finite field of characteristic p and S a set of closed points of X of upper Dirichlet density 1 (resp. > 0). Let $\mathcal{C}$ be an overconvergent F-isocrystal on X. Then the Zariski closure of the union of conjugacy classes of Frobenius elements corresponding to S contains the whole (resp. at least one connected component) of the Tannakian group of $\mathcal{C}$. (When $\mathcal{C}$ is semisimple, this has been proved by Hartl and Pál.) Our proof is done by reducing the problem to the l-adic case for a suitable prime l, via the theory of companions and certain subtle algebraic-group-theoretic arguments. We next discuss a similar Tannakian Chebotarev density theorem for convergent F-isocrystals and obtain various new (partial) results by a purely p-adic method (that is, without resorting to the theory of companions).

Teppei Takamatsu: On Fedder type criteria for Quasi-F-splitting

In algebraic geometry of positive characteristic, singularities defined by the Frobenius map, including the notion of Frobenius-splitting, have played a crucial role. Moreover, there are powerful criteria, so-called Fedder's criteria, to confirm such properties. Yobuko recently introduced the notion of quasi-F-splitting and quasi F-split heights, which generalize and quantify the notion of Frobenius-splitting, and proved that quasi F-split heights coincide with Artin-Mazur heights for Calabi-Yau varieties. In this talk, I will explain the generalization of Fedder's criteria to quasi-F-splitting, and introduce examples and applications of such criteria. This talk is based on a joint paper with Tatsuro Kawakami and Shou Yoshikawa.

Takeshi Tsuji: Prismatic cohomology and q-Dolbeault complex

We give a local description of a prismatic crystal and its cohomology in terms of a q-Higgs module and its q-Dolbeault complex on a bounded prismatic envelope when the base is a prism over $\mathbb{Z}_p[\![q-1]\!]$. Then we discuss a way how to glue the local description by Cech construction, which allows us to compare Ainf-cohomology with coefficients, studied in a joint work with M. Morrow, and prismatic cohomology. We also give a global description when an embedding into a projective space is given.

Nobuo Tsuzuki: Mod 2 Galois representations of even degree and reciprocity

In this talk we study mod 2 representations ρ of arbitrary even degree over number fields K by reciprocity which is a system of mod 2 congruences between trace of ρ and number of solutions of reduction of certain polynomial f over K at almost all finite places of any finite extension of K. We apply our study to the case of Galois representations associated to "mirror symmetry" of Dwork's family of Calabi-Yau varieties of certain dimensions. This is a partially joint work with Takuya Yamauchi.

Kazuki Yamada: Poincaré duality for p-adic Hodge cohomology

I will explain the construction of a compactly supported version of (rigid analytic) Hyodo-Kato cohomology and Hyodo-Kato map for a strictly semistable log scheme which admits a good compactification. As the construction is in accordance with a very simple way of thinking, we immediately see all additional structures and the Hyodo-Kato map are compatible with the Poincaré duality. I will also mention some remarks on the foundation of weak formal scheme theory which we modified recently. This is a joint work with Veronika Ertl.

Fuetaro Yobuko: F-splitting, canonical lifting and Hodge-Wittness.

In this talk, I will talk about two topics related to F-splitting; the first one is "F-splitting, canonical lifting and Cartier transform". Recently, Piotr Achinger and Maciej Zdanowicz constructed a canonical mod p^2-lifting for an F-split scheme and proves that, for an ordinary abelian variety or a K3 surface, their canonical lifting coincides with the mod p^2 of the canonical lifting from the Serre-Tate theory. On the other hand, each mod p^2-lifting defines the Cartier transform by Ogus-Vologodsky's non abelian Hodge theory. In this talk, I will talk about my attempt of constructing the Cartier transform in terms of F-splitting for F-split schemes.
The second topic is "Quasi-F-splitting and Hodge-Wittness". Quasi-F-spitting is a generalization of F-splitting. Ekedahl proves that the product of an ordinary (in the sense of Kato) scheme and a Hodge-Witt scheme is again Hodge-Witt. In this talk, I will explain that an F-split analogue of this statement holds, that is the product of an F-split scheme and a quasi-F-split scheme is quasi-F-split.