Mini-Conference on Special Functions, Geometry and Arithmetic

Date : April 12 (Fri) - 13 (Sat), 2019

Venue : Kawai Hall, Graduate School of Science, Tohoku University

http://www.math.tohoku.ac.jp/~tsuzuki/meeting/2019special/special2019.html

April 12 (Fri)

14:00 – 15:00 **Shigeki Matsuda** (Chiba)

π-exponentials for twisted ramified Witt vectors

I will talk about a generalization of π-exponentials. Pulita's π-exponential series is a generalization of Dwork's exponential series and it plays important roles in the theory of p-adic differential equations. Recently E. J. Pickett and L. Thomas generalized π-exponentials for Lubin-Tate groups over ramified base rings to study integral structures of Galois modules. We extend their constructions further using twisted ramified Witt vectors and show that newly defined π-exponentials also share many properties common with classical π-exponentials. As applications, we determine the radii of convergence of some formal group exponentials and the Dwork-Carliz exponential.

15:15 – 16:15 **Masaaki Yoshida** (Kyushu)

Hyperbolic Schwarz map for Gauss hypergeometric equation

The Schwarz map of the hypergeometric differential equation is found and studied by Schwarz in the beginning of the last century. Since then many kinds of high dimensional versions are studied analytically, algebro-geometrically and arithmetically. But I have had a slight reservation about the definition of the Schwarz map: Its target seems not to be exactly the correct one. I present a correct one, and show how interesting it is.

16:30 – 17:30 **Francesco Baldassarri** (Padova)

Liftings of p-divisible groups to $\mathbb{Z}_p$, the Dieudonné isomorphism, and entire Fourier expansions on $\mathbb{Q}_p$

We describe a power-series $\Psi_p(x) \in \mathbb{Z}[[x]]$ which extends to an entire p-adic analytic function, sends $\mathbb{Q}_p$ to $\mathbb{Z}_p$, and trivializes the addition law of Barsotti-Witt covectors

$$\Psi(x+y) = \Phi(\Psi(x), \Psi(px), \dots ; \Psi(y), \Psi(py), \dots) \, .$$

The generic fiber of the corresponding $\mathbb{Z}_p$-formal group is the usual $\mathbb{Q}_p$-analytic group $\mathbb{G}_a$ while its special fiber is the p-divisible group $(\mathbb{Q}_p/\mathbb{Z}_p)_{\mathbb{F}_p}$. Via the Dieudonné isomorphism to hyperexponential vectors we obtain a family of p-adically entire functions $\{G_q\}_{q \in S}$, bounded on $\mathbb{Q}_p$, where $S = \mathbb{Z}[1/p] \cap \mathbb{R}_{\geq 0}$, such that

$$G_q(x+y) = \sum_{q_1+q_2=q} G_{q_1}(x) G_{q_2}(y) \quad , \quad \forall q \in S \, ,$$

where the sequence of finite partial sums converges uniformly on bounded subsets of $\mathbb{C}_p^2$.

We identify the algebra of functions on the formal perfectoid open unit disc $\mathbb{D}$ over $\mathbb{Z}_p$, namely the (p,T)-adic completion $\mathscr{D}$ of the ring $\mathbb{Z}_{(p)}[T^{1/p^\infty}]$, with the topological Hopf algebra of $\mathbb{Z}_p$-valued *uniform measures* on $\mathbb{Q}_p$ and describe a special element $T = \mu_{\mathrm{can}} \in \mathscr{D}$, called the *canonical measure*.

For any $q \in S$, the measure μ^q_{can} is well-defined in $\mathscr{D}$ and any bounded uniformly continuous function $f : \mathbb{Q}_p \to \mathbb{Q}_p$ admits the generalized Amice-Fourier expansion

$$f = \sum_{q \in S} \Big(\int_{\mathbb{Q}_p} f \, \mu^q_{\text{can}} \Big) \, G_q = \text{``} \sum_{q \in S} f^{[q]}(0) \, G_q \text{''} \; .$$

The convergence of this expansion presents the same problems which arise in classical harmonic analysis.

April 13 (Sat)

10:00 – 11:00 **Nobuo Tsuzuki** (Tohoku)

On a 2-adic model of the family of hypergeometric Calabi-Yau varieties

In this talk we make an attempt to construct a 2-adic model of the family of hypergeometric Calabi-Yau varieties over $\mathbb{Z}[1/2][z, 1/z(1-z)]$ related to the generalized hypergeometric series

$${}_{n+1}F_n \left(\begin{array}{c} 1/2, \ldots, 1/2, \\ 1, \ldots, 1 \end{array} ; z \right) = \sum_{i=0}^{\infty} \left(\frac{(1/2)_i}{i!} \right)^{n+1} z^i.$$

We give examples of K3 surfaces over number fields with everywhere good reduction by specializations of the family.

11:15 – 12:15 **Hironori Shiga** (Chiba)

Deformation of simple K3 singularities and GKZ, alla mandala

The K3 singularity is a 3-dimensional analogue of simple elliptic singularities. By them we can consider several families of lattice polarized K3 surfaces and its periods. Sometimes the family is induced from a reflexive polytope which enables to describe the corresponding GKZ equations. We consider these situations alla mandala.

Sponsor : This conference is supported by JSPS Kakenhi grant (18H03667).

Organizer : Nobuo Tsuzuki (Tohoku)