p-adic cohomology and arithmetic geometry 2019

Abstracts

Piotr Achinger: Canonical liftings and log structures

In the context of mirror symmetry, the moduli space of complex Calabi-Yau varieties acquires canonical local coordinates near a "large complex structure limit point". In characteristic p geometry, the formal deformation space of an ordinary Calabi-Yau variety tends to have such canonical coordinates ("Serre-Tate parameters") as well. As observed e.g. by Jan Stienstra, these situations are formally very similar, and one would like to compare the two when both make sense. A framework for doing this could be supplied by a version of Serre-Tate theory for log Calabi-Yau varieties. In my talk, I will describe a first step in this direction; a construction of canonical liftings modulo p^2 of certain log schemes. I will link this to a question of Keel describing global moduli of maximal log Calabi-Yau pairs.

Konstantin Ardakov: Irreducibility of global sections of Drinfeld line bundles

Let F be a finite extension of $\mathbb{Q}_p$ and let $G = GL_2(F)$. The Drinfeld tower provides a large and interesting family of G-equivariant vector bundles with connection on the p-adic upper half plane over F. When $F = \mathbb{Q}_p$, it is known thanks to the work of Dospinescu and Le Bras that the spaces of global sections of these vector bundles describe the difference between the entire space of locally analytic vectors and the classical space of smooth vectors in the admissible Banach space representations of G associated by Colmez to a 2-dimensional de Rham representation of the Galois group of F. When F is arbitrary but the vector bundle with connection in question is a non-trivial line bundle, we use techniques from the theory of arithmetic $\mathcal{D}$-modules such as Frobenius descent, microdifferential operators and characteristic cycles to prove the irreducibility and coadmissibility of its space of global sections as a module over the locally analytic distribution algebra of G. This is joint work with Simon Wadsley.

Masanori Asakura: A generalization of Ross symbol in higher K-groups and p-adic hypergeometric functions

I give a talk on my recent work (in progress) on regulators and hypergeometric functions. The Ross symbol is defined to be an element $\{1-x, 1-y\}$ in K_2 of a Fermat curve $x^n + y^m = 1$. Ross showed that his symbol is non-trivial and studied the Beilinson conjecture on the special value of the L-functions of the Fermat curves. In this talk I give a generalization of the Ross symbol in K_s of a variety $(1-x_1^{n_1})\ldots(1-x_s^{n_s}) = t$ (I will explain in the talk why this can be seen as a generalization). The main results are

- the Beilinson regulator is described by a hypergeometric function ${}_{s+2}F_{s+1}$, and;
- the p-adic regulator is described by a certain p-adic hypergeometric function (different from Dwork's) introduced by the speaker.

Kenichi Bannai: On the p-adic polylogarithm function for totally real fields

The p-adic polylogarithm functions are p-adic analytic functions defined as p-adic analogs of the classical polylogarithm functions, which are multi-valued holomorphic functions defined by an iterated integral on the projective line minus three points. In this talk, we will construct higher-dimensional analogs of the p-adic polylogarithm functions, which we define as certain higher p-adic cohomology classes of an algebraic torus associated to totally real fields. We will also investigate the relation of these classes to special values of p-adic Hecke L-functions associated to the totally real field.

Daniel Caro: Arithmetic D-modules over Laurent series fields

Let k be a characteristic $p > 0$ field, V be a complete DVR whose residue field is k and fraction field K is of characteristic 0. We denote by $\mathcal{E}_K$ the Amice ring with coefficients in K, and by $\mathcal{E}_K^\dagger$ the bounded Robba ring with coefficients in K. Berthelot's classical theory of Rigid Cohomology over varieties $X/k((t))$ gives $\mathcal{E}_K$-valued objects. Recently, Lazda and Pál developed a refinement of rigid cohomology, i.e. a theory of $\mathcal{E}_K^\dagger$-valued Rigid Cohomology over varieties $X/k((t))$. The purpose of this talk is to introduce to a theory of arithmetic D-modules with $\mathcal{E}_K^\dagger$-valued cohomology which satisfies a formalism of Grothendieck's six operations. We will distinguish the relative case and the absolute case.

Richard Crew: Nilpotent arithmetic D-modules

I will discuss the ideal theory of Berthelot's rings of arithmetic D-modules. A certain completion of the ring of level m arithmetic differential operators gives a natural interpretation of the category of topologically nilpotent D-modules of level m. I will discuss the problem of determining the global dimension of these rings.

Christine Huyghe: Arithmetic D-modules and representations of the crystalline distribution algebra

This is joint work with Tobias Schmidt. Let G be a complex reductive algebraic group, and $\mathfrak{g}$ its Lie algebra. There is an equivalence of categories between the category of D-modules over the flag variety of G and $\mathfrak{g}$-modules with central character. The D-module corresponding to the simple quotient of a Verma module with trivial central character is obtained as an intermediate extension of the constant sheaf of a Bruhat cell. I will explain a p-adic analogue of this result, using recent results of Abe-Caro on arithmetic D-modules. On the way, I will also introduce an arithmetic p-adic category analogous to the classical O-category appearing in classical representation theory of Lie algebras and explain how objects of this category can be localized.

Fumiharu Kato: Toward rigid analytic approach to uniform non-archimedean Pila-Wilkie type theorems

This is work in progress in collaboration with Gal Binyamini (Weizmann Institute). The original Pile-Wilkie inequality, which estimates the number of rational points on (sub-)analytic subsets, has been proven by the methods of interpolation determinants and Yondin-Gromov parametrization. Later, entirely complex analytic approach has been established by Binyamini-Novikov. Clucker-Comte-Loeser and Clucker-Forey-Loeser gave non-archimedean versions, both in 0 and positive characteristics, by model-theoretic argument. We would like to give rigid analytic approach to reprove, and improve their results. We will explain the outline of our approach, together with some new ideas in rigid geometry.

Kiran S. Kedlaya: Etale and crystalline companions (parts 1 and 2)

Let X be a smooth irreducible variety over a finite field of characteristic p. Fix a Weil cohomology theory with algebraically closed coefficients conatining a prescribed algebraic closure of $\mathbb{Q}$ (either ℓ-adic cohomology for some prime ℓ not equal to p, or p-adic rigid cohomology). Deligne's "companion conjecture" then asserts that if one takes every irreducible lisse coefficient with finite determinant and associates to it the tuple of Frobenius characteristic polynomials at all closed points, then the resulting collection of tuples does not depend on the choice of the Weil cohomology theory. By previous work of Deligne, Drinfeld, Abe-Esnault, and the speaker, this is known in all cases except "ℓ to p". We discuss the proof of this implication, i.e., the assertion that etale coefficient objects have crystalline companions.

In part 1, we will review some key steps from the proof of the "p to ℓ" assertion (i.e., the existence of etale companions of crystalline coefficients), including the proof that the "ℓ to p" case can be checked after an alteration. This allows us to reduce to the case of a tame etale coefficient on a fibration in curves.

In part 2, we introduce certain moduli stacks of tame mod-p^n crystals on a fibration in curves. We then explain how some uniformity statements about tame isocrystals (and their lattices) on curves can be used to control the geometry of these moduli stacks, and thereby to construct crystalline companions.

Yukiyoshi Nakajima: The action of the crystalline Weil-Deligne group on the infinitesimal cohomology in mixed characteristics

For a separated scheme of finite type over an algebraic closed field of a complete discrete valuation field of mixed characteristics, we define a canonical action of the crystalline Wei-Delignel group on the infinitesimal cohomology of it and we prove that this action is contravaiantly functorial. This is a generalization of Berthelot-Ogus's theorem for the de Rham cohomology of a proper smooth scheme with potentially good reduction and this proves a conjecture of Ogus in a generalized form.

Shusuke Otabe: Embedding problems for stacky curves in positive characteristic

In 1994, Raynaud and Harbater settled Abhyankar's conjecture, which describes all the finite quotients of Grothendieck's étale fundamental group of an affine smooth curve in positive characteristic. By independent works due to Harbater and Pop, finer results were also obtained, which particularly control the ramifications of the Galois covers. In this talk, I will consider a purely inseparable counterpart of the refined versions of the conjecture, formulated in terms of Nori's fundamental group schemes of root stacks, and discuss it for finite connected solvable group schemes.

Andrea Pulita: Cohomology with compact supports of differential equations on smooth Berkovich curves

In some recent papers by myself and J. Poineau we obtained necessary and sufficient conditions for the finite dimensionality of the cohomology of a certain class of differential equations over a general smooth Berkovich curve (that is vector bundles with a connection, either in the analytic or meromorphic contexts). This class of equations is actually large enough to include any differential equation on a smooth Berkovich curve that has ever been studied. Indeed, the only assumption on these equations concerns a certain Liouville condition at the boundary of the curve. For instance, if the residual field has characteristic 0, then the Liouville condition is empty and the class consists of all possible differential equations. We discovered that the most part of these equations have infinitely dimensional de Rham cohomology even over very kind curves such a s disk or annuli. The condition ensuring the finite dimensionality concerns the behavior of the radii of convergence of the solutions at the boundary of the curve. In this talk I will recall these results and at the end I will give an idea of a work in progress with R. Crew about the finite dimensionality of the cohomology with compact supports.

Takeshi Tsuji: Coefficients in integral p-adic Hodge theory via generalized A_{inf}-representations

This is joint work with Matthew Morrow on a theory of coefficients in the integral p-adic Hodge theory via A_{inf} cohomology by Bhatt, Morrow, and Scholze. In this talk, I will focus on the local theory. I will explain how a generalized A_{inf}-representation (trivial modulo μ) with Frobenius gives rise to a $\mathbb{Z}_p$-representation, a module with integrable connection, an F-crystal on the special fiber, and a filtered F-crystal over the base A_{crys}, and then how their cohomology groups are related with the local A_{inf} cohomology. By following an analogy of the descent used in the p-adic Simpson correspondence by Faltings, we see that, given a choice of coordinates, a generalized A_{inf}-representation with Frobenius corresponds to a module with integrable q-connection, and then to a module with integrable q-Higgs field (both with Frobenius structure). This is the key to the above construction and to local explicit computation of the local A_{inf} cohomology.

Masha Vlasenko: Dwork crystals

In his work on zeta functions for families of algebraic varieties Bernard Dwork discovered a number of remarkable p-adic congruences. In this lecture I present our recent work with Frits Beukers which explains the underlying mechanism of such congruences.

John Welliaveetil: Constructibility in non-Archimedean geometry

The theory of adic spaces was developed by R. Huber to better understand the étale cohomology of Rigid analytic spaces. Despite the many applications of the theory, we are yet to find a notion of constructible sheaves which is well behaved with respect to the six functor formalism. In this talk, we propose a theory of constructibility by exploiting the nearby cycles functor associated to a certain class of adic spaces and study the extent to which this notion respects the six functor formalism.

Takao Yamazaki: Lax tensor products for reciprocity sheaves

Voevodsky's category of homotopy invariant sheaves with transfers is equipped with a highly non-trivial tensor structure. We explain our attempt to generalize it to the category of reciprocity sheaves, which is strictly larger than Voevodsky's one. It is applied to compute the Chow groups with modulus of a product of varieties. (Joint work with Kay Ruelling and Rin Sugiyama.)

Seidai Yasuda: Integral structures of two dimensional crystalline representations

Let V be a two dimensional crystalline representation of the absolute Galois group of $\mathbb{Q}_p$, where p is an odd prime. In this talk, we discuss the structure of Galois stable integral lattices of V when the difference of Hodge-Tate weights are not too large. The material of the talk is based on my joint work with Go Yamashita.