

REVERSE MATHEMATICS AND NONSTANDARD ANALYSIS

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ABSTRACT

By now, it is well-known that large parts of ‘ordinary’ mathematics can be developed in systems much weaker than ZFC ([6], [7]). However, most theories under consideration are at least as strong as WKL_0 , which is conservative over $I\Sigma_1$. It is usually mentioned (see e.g. [1], [2] and [6]) that it should be possible to develop a large part of mathematics in much weaker systems, in particular in $I\Delta_0 + \text{exp}$ and related systems. Most notably, there is Friedman’s Grand Conjecture (see [2] and [3]):

Every theorem published in the Annals of Mathematics whose statement involves only finitary mathematical objects (i.e. what logicians call an arithmetical statement) can be proved in EFA.

In 1929, Jacques Herbrand already made a similar claim, but without specifying the underlying logical system (see [4, p152]).

In this talk, we discuss the reverse mathematics of ERNA, which is a nonstandard version of $I\Delta_0 + \text{exp}$ (see [5]). As it turns out, we can obtain, in ERNA, a copy ‘up to infinitesimals’ of Reverse mathematics for WKL_0 . The principle corresponding to Weak König’s Lemma is a transfer principle and generalizations follow from an elegant bootstrapping argument. We briefly discuss the counterpart of ACA_0 in this setting. If time permits, we will discuss the boundaries and limitations of our approach.

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