Monotonicity and Rigidity of the $\mathcal{W}$-entropy on $\mathsf{RCD}^*(0, N)$ spaces

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Perelman's $\mathcal{W}$-entropy plays a crucial role in his seminal work on Ricci flow. It is well-known by Perelman's entropy formula that the $\mathcal{W}$-entropy is non-increasing in time and a time derivative vanishes if and only if the space is isomorphic to a gradient shrinking Ricci soliton. L. Ni brought the notion of $\mathcal{W}$-entropy to time-homogeneous Riemannian manifolds, and the corresponding results has been studied in the literature under nonnegative Ricci curvature in an appropriate sense.

In this talk, we consider the corresponding problem on $\mathsf{RCD}^*(0, N)$ metric measure spaces. $\mathsf{RCD}^*(0, N)$ space is a class of infinitesimally Hilbertian metric measure spaces with the nonnegative Ricci curvature and the upper bound of dimension by N, defined in terms of optimal transport. It includes all (weighted) Riemannian manifold (or smooth metric measure spaces) with nonnegative N-Bakry-Émery Ricci tensor, and Ricci limit spaces with an appropriate curvature-dimension bound. By following Topping's approach to this problem by optimal transport, we prove the monotonicity of the $\mathcal{W}$-entropy *without* deriving the entropy formula. Moreover, we also show a rigidity of this monotonicity. Unlike the smooth case, some other singular spaces than Euclidean spaces admit a vanishing time derivative of the $\mathcal{W}$-entropy. Our result is new even on a weighted Riemannian manifold in the sense that we require no additional bounded geometry assumption which is used to derive the entropy formula. This is a joint work with Xiang-Dong Li (Chinese Academy of Science).

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