Optimal transportation costs of heat distributions in stochastic geometric analysis

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On a complete Riemannian manifold, a contraction property of the Wasserstein distance between heat distribution is known to be equivalent to the presence of a lower Ricci curvature bound. The L^p-Wasserstein distance is the optimal transportation cost of two probability measures whose cost function is given by p-th power of the distance function. In this talk, we consider three extensions of this relation in different directions mainly based on a coupling method of Brownian motions. The first one is the monotonicity of a transportation cost associated with the Perelman's $\mathcal{L}$-functional on a backward Ricci flow. This is an extension of Topping's result by stochastic analysis. The second one deals with a modified transportation cost associated with the total variation between heat distributions on spaceforms. It yields a comparison theorem for the total variation. The third one is a new contraction property for the L^p-Wasserstein distance which is equivalent to the curvature-dimension condition, that is, the combination of an upper dimension bound and a lower Ricci curvature bound.