Identification of heat flows

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It has been known that there are two different ways to formulate the heat distribution on a Euclidean space as gradient flow of a potential functional. One way is as gradient flow of Dirichlet energy in L^2-space of functions and the other is as gradient flow of relative entropy in L^2-Wasserstein space of probability measures. Those gradient flows can be defined even on a singular space where a usual differential calculus is not available. Since the identification of those gradient flows in the framework of a Euclidean space heavily relied on a (second order) differentiable structure of the space, it was totally nontrivial question whether those two gradient flow coincides or not on such a singular space.

In this talk, we will give an overview of a recently developed approach for identifying these two gradient flows, which works even on abstract metric measure spaces. As a consequence of our identification, we can combine known properties of those flows to deduce the further properties of the heat flow. In this way, we obtain the Lipschitz continuity of the heat kernel as well as Bakry-Émery's gradient estimate and the Γ_2-condition under an appropriate lower curvature bound of the underlying space. A part of this talk is based on a joint work with N. Gigli (Nice) and S. Ohta (Kyoto).