

# AN EXTENSION OF WASSERSTEIN CONTRACTION ASSOCIATED WITH THE CURVATURE-DIMENSION CONDITION

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We obtain a new characterization of complete Riemannian manifolds with lower Ricci curvature bound and upper dimension bound in terms of the Wasserstein distance between heat distributions. It is formulated as a local space-time Lipschitz estimate of the Wasserstein distance between two heat distributions with different initial data at different times. It extends a part of result in [2] where they studied the case that no upper dimension bound is imposed. The proof is based on establishing an equivalence with a gradient estimate of heat semigroups studied in [3], by following a strategy in [1]. In addition, we can obtain a sharper estimate by using a coupling method.

## REFERENCES

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