Duality results on gradient estimates and Wasserstein controls

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Under mild assumptions, I will state an equivalence between the following two different kinds of inequalities for a Markov kernel:

1. Lipschitz type estimate with respect to the L^p-Wasserstein distance,

2. Bakry-Émery type L^q-gradient estimate.

Typically, both of them have been used to characterize the rate of convergence of (diffusion) semigroups and these estimates somehow reflect the geometry behind them. For instance, for the heat semigroup on a complete Riemannian manifold, a special form of them was known to be equivalent by means of employing one more equivalent condition, the presence of a lower Ricci curvature bound. In particular, these estimates exhibit an exponential rate of convergence to the equilibrium state when the curvature is strictly positive. Our duality result enables us to recover it without relying on any notion of curvatures.

As an application, a Wasserstein control of hypoelliptic diffusions on Lie groups will be derived from a known gradient estimate of Bakry-Émery type. Potentially, our duality result could be applicable in various framework beyond the above-mentioned cases, even on an infinite dimensional state space.