Splitting of singular fibers and vanishing cycles

Takayuki OKUDA
the University of Tokyo

Kanazawa University
Satellite Plaza
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Degenerations and their splitting deformations
Degeneration of Riemann surfaces

\( M \): a smooth complex surface \hspace{1cm} \( \Delta \): the unit disk in \( \mathbb{C} \)
\( \pi : M \to \Delta \): a proper surjective holomorphic map s.t.

- \( X_s := \pi^{-1}(s) \ (s \neq 0) \) are smooth curves of genus \( g \).
- \( X_0 := \pi^{-1}(0) \) is a singular fiber.

\( 0 \) is a unique critical value.

\( \pi : M \to \Delta \) is called a degeneration (or, degenerating family) of Riemann surfaces of genus \( g \).

Regard \( X_0 \) as the divisor defined by
\[
X_0 = \sum m_i \Theta_i,
\]
where \( \Theta_i \) is an irreducible component with multiplicity \( m_i \).
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Splitting of singular fibers

\( \pi : M \to \Delta \) : a degeneration with singular fiber \( X_0 \)

\( \{ \pi_t : M_t \to \Delta \} \) : a family of deformations of \( \pi : M \to \Delta \)
i.e. \( \pi_0 : M_0 \to \Delta \) coincides with \( \pi : M \to \Delta \).

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\text{If } \pi_t (t \neq 0) \text{ has } k \text{ singular fibers } X_{s_1}, \ldots, X_{s_k}, k \geq 2, \\
\text{We say that } X_0 \text{ splits into } X_{s_1}, \ldots, X_{s_k}. \]
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How to construct splittings

- **Double covering method**
  - Moishezon (genus 1 case), Horikawa (genus 2 case), Arakawa-Ashikaga (hyperelliptic case)

- **Barking deformation**
  - Takamura (some criterion)

**Fact** (Atoms of singular fibers)
(1) A Lefschetz fiber and (2) a multiple smooth fiber admit no splittings (i.e. any deformation is equisingular).

**Conjecture** (from a topological viewpoint)
Every singular fiber can split into singular fibers each of which is (1) or (2), in finite steps of deformations.
Splittability of singular fibers

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Topological classification

\[ \pi : M \to \Delta \] is topologically equivalent to another degeneration \( \pi' : M' \to \Delta \)

\[ \iff \exists \text{ ori. preserving homeomorphisms} \]

\[ \begin{align*}
H : M &\to M', \\
h : \Delta &\to \Delta \quad \text{s.t.} \quad h \circ \pi = \pi' \circ H.
\end{align*} \]

**Theorem** (Terasoma)

Top. equivalent degenerations are deformation equivalent.

The top. classes of degenerations are completely determined by their topological monodromies.

**Theorem** (Imayoshi, Shiga-Tanigawa, Earle-Sipe)

Every topological monodromy is pseudo-periodic of negative twist.
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Theorem (Matsumoto-Montesinos, 91/92)

\[
\begin{align*}
\{ \text{top. equiv. classes of minimal degenerations of Riemann surfs. of genus } g \} & \quad \overset{1:1}{\leftrightarrow} \quad \{ \text{conj. classes in } \text{MCG}_g \text{ of pseudo-periodic mapp. classes of negative twist} \} \\
\text{via topological monodromy, for } g \geq 2.
\end{align*}
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\end{align*}
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via \textbf{topological monodromy}, for \( g \geq 2 \).

\[\text{Multiple smooth fiber}\]

\[\text{Periodic mapping class w/o multiple points}\]
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Topological classification

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**Singular fiber**

Smooth complex surface \( M \)

**Pseudo-periodic mapping class of negative twist**

Open disk \( \Delta \)
Splittability into Lefschetz fibers
A propeller surface is a Riemann surface $\Sigma_g$ of genus $g \geq 2$ equipped with $\mathbb{Z}_g$-action s.t. $\Sigma_g/\mathbb{Z}_g$ has genus 1.

$\omega_g$: a propeller automorphism

$X_g$: the singular fiber with monodromy $\omega_g$
Theorem (Y. Matsumoto)

$\pi : M \to \Delta :$ the degeneration of Riemann surfaces of genus 2 with monodromy $\omega_2$

Then its singular fiber $X_2$ can split into four Lefschetz fibers.

Moreover, their vanishing cycles are as depicted below.
**Theorem (Y. Matsumoto)**

\[ \pi : M \to \Delta : \text{the degeneration of Riemann surfaces of genus 2} \]

with monodromy \( \omega_2 \)

Then its singular fiber \( X_2 \) can split into four Lefschetz fibers. Moreover, their vanishing cycles are as depicted below.
Psudo-propeller maps

$\omega_{3}^{(2)}$

$\gamma$ : a separating simple loop on $\Sigma_g$

s.t. $\Sigma_g \setminus \gamma = \Sigma_{m,1} \sqcup \Sigma_{n,1}$ \quad ($g = m + n, m \geq 1, n \geq 0$)

$\omega_{m}^{(n)}$ : a pseudo-periodic map satisfying

- $\omega_{m}^{(n)} \mid_{\Sigma_{m,1}} \sim \text{a periodic map with a fixed pt of rot angle } \frac{2\pi}{m}$.
- $\omega_{m}^{(n)} \mid_{\Sigma_{n,1}} \sim \text{id.}$

$X_{3}^{(2)}$

NOTE: $(\omega_{m}^{(n)})^m = \tau_{\gamma}$

$X_{m}^{(n)}$ : the singular fiber with monodromy $\omega_{m}^{(n)}$

$n = 2$

$m = 3$
Psuedo-propeller maps

\[ \omega^{(2)}_3 \]

\[ id \sim \gamma \]

\[ n = 2 \]

\[ m = 3 \]

\[ m = 3 \]

\[ X^{(2)}_3 \]

\[ \{ n = 2 \} \]

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\[ \omega^{(n)}_m \mid_{\Sigma_{n,1}} \sim \text{id.} \]

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\[ X^{(n)}_m : \text{the singular fiber with monodromy } \omega^{(n)}_m \]
Psudo-propeller maps

$X_4^{(1)}$

$X_5^{(0)}$

$\omega_4^{(1)}$

$\omega_5^{(0)} = \omega_5$

$\gamma$
Psudo-propeller maps

\[ X^{(3)}_2 \]

\[ X^{(4)}_1 \]

\[ \omega^{(3)}_2 \]

\[ \omega^{(4)}_1 = \tau_\gamma \]
Theorem (O-Takamura)

1. For any $m \geq 2$, $n \geq 0$, the singular fiber $X_m^{(n)}$ can split into $X_{m-1}^{(n+1)}$ and three Lefschetz fibers.

2. For any $g \geq 2$, we have the following sequence:

\[ X_g^{(0)} \rightarrow X_{g-1}^{(1)} \rightarrow \cdots \rightarrow X_{2}^{(g-2)} \rightarrow X_{1}^{(g-1)}, \]

where “$A \rightarrow B$” means “$A$ splits into $B$ and 3 Lefschetz fibers.”
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\[\begin{array}{c}
X_m^{(n)} \\
\begin{array}{c}
\{ \begin{array}{c}
m \\
1
\end{array} \end{array}
\end{array} \rightarrow \begin{array}{c}
X_{m-1}^{(n+1)} \\
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\[
\begin{align*}
\{ \ &m \ 
\} \ n \ 
\end{align*}
\]

\[
\begin{align*}
\{ \ &m-1 \ 
\} \ n+1
\end{align*}
\]
\[ X_m^{(n)} \rightarrow X_{m-1}^{(n+1)} \]

\[ g = 2 \]

\[ g = 3 \]

\[ g = 4 \]
\[ X_m^{(n)} \rightarrow X_{m-1}^{(n+1)} \]

\[
\begin{align*}
g &= 2 \\
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\]
Results

\[ X_m^{(n)} \rightarrow X_m^{(n+1)} \]

\[ g = 2 \]

\[ g = 3 \]

\[ g = 4 \]
$$X^{(n)}_m \rightarrow X^{(n+1)}_{m-1}$$

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\[ g = 3 \]

\[ g = 4 \]
$$X_m^{(n)} \rightarrow X_m^{(n+1)}$$

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Results

$X_m^{(n)} \rightarrow X_{m-1}^{(n+1)}$

$g = 2$

$g = 3$

$g = 4$
Remark of Theorem

1. Generalization of Matsumoto’s splitting for genus 2.
2. Analogous to adjacency diagrams of singularities:

\[ A_6 \rightarrow A_5 \rightarrow A_4 \rightarrow A_3 \rightarrow A_2 \rightarrow A_1 \]

\[ D_6 \rightarrow D_5 \rightarrow D_4 \]

\[ E_6 \]

3. A splitting of a singular fiber into Lefschetz fibers gives a **Dehn-twist expression** of its topological monodromy.
Case $g = 2$ (bis)

Theorem (Y. Matsumoto)

1. The singular fiber $X_2$ can split into four Lefschetz fibers, and their vanishing cycles are as depicted below.

2. $\omega_2 = \tau_0 \circ \tau_a \circ \tau_b \circ \tau_c$. 

![Diagram](image-url)
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\[ \omega_2 \]

[Diagram of singular fiber with labeled cycles]
Proposition

1. $X_3^{(0)}$ can split into $X_2^{(1)}$ and three Lefshetz fibers, and their vanishing cycles are as depicted below.

2. $\omega_3 = \omega_2^{(1)} \circ \tau_{a_1} \circ \tau_{b_1} \circ \tau_{c_1}$. 
Case $g = 3$

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[Diagram of vanishing cycles and Lefshetz fibers]
Proposition

1. $X_2^{(1)}$ can split into four Lefshetz fibers (including $X_1^{(2)}$), and their vanishing cycles are as depicted below.

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**Proposition**

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Case $g = 3$

**Proposition**

$$\omega_3 = \tau_0 \circ (\tau_{a_2} \circ \tau_{b_2} \circ \tau_{c_2}) \circ (\tau_{a_1} \circ \tau_{b_1} \circ \tau_{c_1}).$$
Theorem

\[ \omega_g = \tau_0 \circ (\tau_{a_{g-1}} \circ \tau_{b_{g-1}} \circ \tau_{c_{g-1}}) \circ \cdots \circ (\tau_{a_1} \circ \tau_{b_1} \circ \tau_{c_1}). \]
\( f \in \text{MCG}(\Sigma_g) : \) a periodic mapping class of order \( m \)

\[
b(f) := \# \{ \text{branch points of } \Sigma_g \to \Sigma_g/f \}
\]

\[
p(f) := \# \{ \text{propeller points of } f \} \leq h(f)
\]

\[
r(f) := \sum q_j/\ell_j \in \mathbb{Z}_+ : \text{the total valency sum}
\]

**Theorem (O)**

\( X \) : the singular fiber equipped with periodic monodromy \( f \)

Suppose \( f \) satisfies at least one of the following:

- \( b(f) - p(f) \leq r(f) \).
- \( b(f) - p(f) \leq 2, \ r(f) = 1 \) and \( \text{genus}(\Sigma_g/f) = 0 \).

Then \( X \) can splits into Lefschetz fibers.
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Then \( X \) can splits into Lefschetz fibers.
**Remark of Theorem**

1. If $m = 2, 3$, then $X$ can split into Lefschetz fibers.

2. | genus | 1 | 2 | 3 | 4 | 5 | 6 |
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<tr>
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<td>47</td>
<td>72</td>
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<tr>
<td># of periodic m.c. satisfying (*)&amp;</td>
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<td>14</td>
<td>30</td>
<td>41</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td># of powers of periodic m.c. satisfying (*)&amp;</td>
<td>8</td>
<td>16</td>
<td>45</td>
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Thank you for your attention.