

FJV2017 Kagoshima

The 5th Franco-Japanese-Vietnamese Symposium on Singularities

27 - 28 Oct. (mini-school), 29 Oct. - 3 Nov. (symposium)

Kagoshima University

(Place: Room 101, Graduate School of Science and Engineering)

Mini-School

27th October, Friday

9:00 - 9:50 Wojciech Kucharz (Jagiellonian University)

On continuous rational functions, I

10:00-10:50 Huy Vui Ha (Institute of Mathematics, VAST)

Lojasiewicz inequality – Smooth and global versions, I

11:10-12:00 Ilia Itenberg (Université Pierre et Marie Curie, Paris 6)

Tropical homology and topology of real algebraic varieties, I

14:00-14:50 Wojciech Kucharz (Jagiellonian University)

On continuous rational functions, II

15:00-15:50 Huy Vui Ha (Institute of Mathematics, VAST)

Lojasiewicz inequality – Smooth and global versions, II

16:10-17:00 Ilia Itenberg (Université Pierre et Marie Curie, Paris 6)

Tropical homology and topology of real algebraic varieties, II

28th October, Saturday

9:00 - 9:50 Wojciech Kucharz (Jagiellonian University)

On continuous rational functions, III

10:00-10:50 Huy Vui Ha (Institute of Mathematics, VAST)

Lojasiewicz inequality – Smooth and global versions, III

11:10-12:00 Ilia Itenberg (Université Pierre et Marie Curie, Paris 6)

Tropical homology and topology of real algebraic varieties, III

Wojciech Kucharz (Jagiellonian University)

On continuous rational functions

Abstract: Let X be an algebraic subset of \mathbb{R}^n and let $f: W \rightarrow \mathbb{R}$ be a function defined on some subset $W \subset X$. We say that f is *regular at a point* $x \in W$ if there exist two polynomial functions p, q on \mathbb{R}^n such that $q(x) \neq 0$ and $f = p/q$ on $W \cap \{q \neq 0\}$. We say that f is *regular on a subset* $A \subset W$ if it is regular at each point of A . Let Y be the Zariski closure of W in X . Any rational function R on Y determines a regular function $R: Y \setminus \text{Pole}(R) \rightarrow \mathbb{R}$, where $\text{Pole}(R)$ stands for the polar set of R . We say that R is a *rational representation of f* if there exists a Zariski open dense subset $Y^{\circ} \subset Y \setminus \text{Pole}(R)$ such that $f|_{W \cap Y^{\circ}} = R|_{W \cap Y^{\circ}}$. It turns out that functions having rational representations enjoy some remarkable properties (especially if they are continuous in the Euclidean topology). I will discuss some results obtained recently by G. Fichou, J. Huisman, J. Kollár, W. Kucharz, K. Kurdyka, F. Mangolte, J.P. Monnier, and K. Nowak. Furthermore, I will briefly describe applications of such functions to topological problems in real algebraic geometry.

Huy Vui Ha (Institute of Mathematics, VAST)

Lojasiewicz inequality – Smooth and global versions

Abstract: In these lectures we consider smooth and global versions of the classical Lojasiewicz inequality. The relationship between the global Lojasiewicz inequality and atypical values at infinity of real polynomials are investigated.

Ilya Itenberg (Université Pierre et Marie Curie, Paris 6)

Tropical homology and topology of real algebraic varieties

Abstract: Tropical geometry can be seen as an algebraic geometry based on tropical numbers. The role of algebraic varieties in tropical geometry is played by certain piecewise-linear objects, called tropical varieties. One of the sources of tropical geometry is topology of real and complex algebraic varieties, and in particular, the combinatorial patchworking construction, which provides an approximation of some tropical varieties by one-parametric families of complex varieties. The purpose of these lectures is to present basic notions of tropical geometry and to discuss tropical homology together with its relations to Hodge decompositions (respectively, homology) in complex (respectively, real) world

FJV2017 Kagoshima: Symposium

29 Oct. - 3 Nov., Kagoshima University

(Place: Room 101, Graduate School of Science and Engineering)

29th October, Sunday

- 13:30-13:50 Registration
- 13:50-14:00 Opening address
- 14:00-14:50 Shihoko Ishii (Tokyo Woman's Christian University)
The Conjecture of Boundedness of number of blowing-ups to compute MJ-minimal log discrepancies in arbitrary characteristic
- 15:00-15:30 Cong Trinh Le (Qui Nhon University)
Equinormalizable deformations of isolated non-normal singularities over normal base spaces
- 16:00-16:50 Jean-Philippe Monnier (Université d'Angers)
Weak normalization and seminormalization in real algebraic geometry

30th October, Monday

- 9:30-10:20 Naohiko Kasuya (Kyoto Sangyo University)
Knots and links of complex tangents
- 10:30-11:00 Pauline Bailet (Universität Bremen)
A vanishing result for the first twisted cohomology of affine varieties and applications to line arrangements
- 11:20-12:10 Duc Tai Trinh (Dalat University)
Resurgent asymptotics of the cubic model
- 14:00-14:50 Susumu Tanabé (Galatasaray University)
Monodromy of period integrals associated to hypersurfaces in toric variety via Mellin-Barnes integrals
- 15:00-15:30 Guillaume Rond (Aix-Marseille Université)
Algebraization and equisingular deformations
- 16:00-16:50 Todor Milanov (IPMU, University of Tokyo)
The period map for quantum cohomology of P^2

31st October, Tuesday

- 9:00 - 9:50 Christophe Eyrat (Polish Academy of Sciences)
Uniform stable radius, Lê numbers and topological triviality for line singularities
- 10:00-10:30 Thi Thao Nguyen (Hanoi National University of Education)
Newton polygon and the number of integer points in some semialgebraic sets
- 10:50-11:40 Kazushi Ueda (University of Tokyo)
Gamma conjecture for Brieskorn-Pham singularities
- 11:50-12:50 **Poster Session**
- Van Doat Dang (Thang Long High School)
Computation Lojasiewicz exponents of the global Lojasiewicz inequality (work with Huy Vui Ha)
- Weili Guo (Hokkaido University)
On the Falk Invariant of Hyperplane Arrangements
- Kazumasa Inaba (Tohoku University)
Topology of the Milnor fibrations of polar weighted homogeneous polynomials
- Phien Phan (University of Khanh Hoa, Nha Trang)
A quantitative form of the splitting lemma
- Delphine Pol (Université d'Angers)
Free singularities
- Homare Tadano (Tokyo University of Science)
Compactness Theorems, Gap Theorems, and Diameter Bounds for Sasaki Manifolds
- Takuhiro Takahashi (Tohoku University)
Tropicalization of 1-tacnodal curves on toric surfaces
- Nhat Tan Tran (Hokkaido University)
G-Tutte polynomials and application to topology of subgroup arrangements
- Asahi Tsuchida (Hokkaido University)
Singularities of bundle homomorphisms between a tangent distribution and a vector bundle
- Dominik Wrazidlo (Kyushu University)
The Milnor 7-sphere does not admit a special generic map into \mathbb{R}^3

1st November, Wednesday

- 9:30-10:20 Adam Parusinski (Université Nice Sophia Antipolis)
Arc-wise analytic equisingularity
- 10:30-11:00 Tat Thang Nguyen (Institute of Mathematics, VAST)
Invariance of the global monodromies in families of polynomials on algebraic varieties and Newton polyhedra
- 11:20-12:10 Takehiko Yasuda (Osaka University)
Equisingularities and dualities of local field masses
- 14:00-14:50 Vincent Grandjean (Universidade Federal do Ceara)
Reduction of regular families of normal operators
- 15:00-15:30 Maria R. Gonzalez-Dorrego (Universidad Autonoma de Madrid)
On singular varieties with smooth subvarieties
- 16:00-16:50 Tatsuo Suwa (Hokkaido University)
Relative Dolbeault cohomology and Sato hyperfunctions
- 18:00-20:00 **Conference dinner**

2nd November, Thursday

- 9:30-10:20 Tuan Hiep Dang (National Center for Theoretical Sciences, Taiwan)
The localization technique and its application to Schubert calculus
- 10:30-11:00 Yimu Yin (Los Angeles)
Bounded motivic integral and motivic Milnor Fiber
- 11:20-12:10 Takuro Abe (Kyushu University)
Hyperplane arrangements and Hessenberg varieties
- 14:00-14:50 Anne Pichon (Aix-Marseille Université)
Lipschitz normal embedding of complex surface singularities
- 15:00-15:30 Zhenjian Wang (Université Cote d'Azur)
On deformations of nodal hypersurfaces
- 16:00-16:50 Kyoji Saito (IPMU, University of Tokyo)
Period domain of for primitive forms for uni-modular exceptional singularities

3rd November, Friday

Free discussion

Shihoko Ishii (Tokyo Woman's Christian University)

The Conjecture of Boundedness of number of blowing-ups to compute MJ-minimal log discrepancies in arbitrary characteristic

Abstract: For a variety defined over a field of positive characteristic, most basic properties are not yet proved: for example openness of the locus of log canonical / canonical / MJ-log canonical / MJ-canonical singularities; also stability of these singularities under a small deformation. These are proved over a field of characteristic 0 by using resolution of the singularities. In this talk, for MJ-log canonical / MJ-canonical singularities, the conjecture about jet schemes which is equivalent to the conjecture in the title induces these basic properties. We also show that the conjecture holds for surface case and non-degenerate hypersurface case in arbitrary characteristic.

Cong Trinh Le (Qui Nhon University)

Equinormalizable deformations of isolated non-normal singularities over normal base spaces

Abstract: In this talk I will present the general theory of equinormalizable deformations of isolated non-normal singularities (INNS). After introducing some open problems and conjectures proposed by G.-M. Greuel in his long new paper [1], I concentrate to solve one of his conjecture (Conjecture 7) for deformations of isolated (not necessarily reduced) curve singularities over normal base spaces.

Reference:

[1] G.-M. Greuel, Equisingular and Equinormalizable Deformations of Isolated Non-Normal Singularities (preprint, 2017) arXiv: 1702.05505.

Jean-Philippe Monnier (Université d'Angers)

Weak normalization and seminormalization in real algebraic geometry

Abstract: We introduce the weak normalization and the seminormalization of a central real algebraic variety. We provide several algebraic and geometric characterizations of these varieties. The study is related to the properties of the rings of continuous rational functions and regulous functions on real algebraic varieties. It is a joint work with G. Fichou and R. Quarez.

Naohiko Kasuya (Kyoto Sangyo University)

Knots and links of complex tangents

Abstract: For an immersion f of a smooth manifold M into \mathbb{C}^n , a point $x \in M$ is said to be a complex tangent if the tangent space $df_x(T_x M)$ contains a complex line. We show that a link in a closed orientable 3-manifold can be realized as the set of complex tangents of a smooth embedding of the 3-manifold into \mathbb{C}^3 if and only if it represents the trivial integral homology class in the 3-manifold. In the proof, we use Saeki's theorem on the topology of the singular sets of stable maps. This is a joint work with Masamichi Takase.

Pauline Bailet (Universität Bremen)

A vanishing result for the first twisted cohomology of affine varieties and applications to line arrangements (Joint work with A. Dimca and M. Yoshinaga)

Abstract: Let S be a smooth proper complex variety of dimension ≥ 2 and $D = \sum_{i=1}^n D_i$ a divisor on S (D_i irreducible). Consider a rank one local system \mathcal{L} on $U = S \setminus D$, with monodromy $t_i \in \mathbb{C}^\times$ around D_i . We give a general vanishing result for the first twisted cohomology group $H_1(U, \mathcal{L})$, generalizing a result due to Cohen-Dimca-Orlik. Then we give some applications in the context of hyperplane arrangement, namely local system cohomology of line arrangement complements. In particular, we will apply our result to determine the monodromy action on the Milnor fiber of two hyperplane arrangements: the Ceva arrangement and the exceptional reflection arrangement of type G_{31} .

Duc Tai Trinh (Dalat University)

Resurgent asymptotics of the cubic model

Abstract: In this talk, I will firstly give a short introduction into Ecalle's theory of resurgent functions. Secondly, I will discuss the resurgent nature of the cubic model, which is motivated from Airy, Weber and quartic models. These models have been studied intensively by A.Voros, F.Pham, E.Delabaere,... Finally, some of recent advances on the topic will be mentioned.

Susumu Tanabé (Galatasaray University)

Monodromy of period integrals associated to hypersurfaces in toric variety via Mellin-Barnes integrals

Abstract: We will discuss a complex analytic approach to obtain a global monodromy representation for period integrals associated to varieties defined in toric variety. We interpret period integrals as GKZ A-hypergeometric functions. Our method relies on

presentation of hypergeometric solution basis obtained by J. Stienstra who generalized the technique used before by A.Givental, S.T.Yau and collaborators. This method establishes correspondence between Stanley-Reisner ring of a polytope and the solution space of A-hypergeometric functions. Further we present a monodromy calculation method by P.R.Horja based on the analytic continuation of Mellin-Barnes integral representation of the GKZ hypergeometric functions. The essential step of analytic continuation procedure consists in a trick named “Mellin-Barnes contour throw.” Several examples for Calabi-Yau varieties will be given.

Guillaume Rond (Aix-Marseille Université)

Algebraization and equisingular deformations

Abstract: We will review some results concerning the algebraization of germs of analytic sets via the construction of equisingular deformations. We will also explain how to construct in an effective way, for a given complex algebraic set X , an algebraic set defined over the algebraic numbers that is homeomorphic to X .

Todor Milanov (IPMU, University of Tokyo)

The period map for quantum cohomology of P^2

Abstract: For a given semi-simple Frobenius manifold we define a period map and propose the problem of finding its inverse. Our definition is motivated from mirror symmetry and the applications that we have in mind are related to reconstructing Gromov—Witten invariants in terms of the monodromy data associated with quantum cohomology. In the first part of my talk, I would like to explain the general settings, while in the second part my plan is to explain the case of quantum cohomology of P^2 . Our main result says that the inverse of the period map can be described in terms of the Eisenstein series E_4 and E_6 .

Christophe Eyrat (Polish Academy of Sciences)

Uniform stable radius, Lê numbers and topological triviality for line singularities

Abstract: Let $\{f_t\}$ be a family of complex polynomial functions with line singularities. We show that if $\{f_t\}$ has a uniform stable radius (for the corresponding Milnor fibrations), then the Lê numbers of the functions $\{f_t\}$ are independent of t for all small t . A similar assertion was proved by M. Oka and D. B. O'Shea in the case of isolated singularities --- a case for which the only nonzero Lê number coincides with the Milnor number.

By combining our result with a theorem of J. Fernández de Bobadilla, we conclude that a family of line singularities in \mathbb{C}^n , $n \geq 5$, is topologically trivial if it has a uniform stable radius.

As an important example, we show that families of weighted homogeneous line singularities have a uniform stable radius if the nearby fibres $f_t^{-1}(\eta)$, $\eta \neq 0$, are “uniformly” nonsingular with respect to the deformation parameter t .

Thi Thao Nguyen (Hanoi National University of Education)

Newton polygon and the number of integer points in some semialgebraic sets

Abstract: Let $f(x, y)$ be a polynomial in two variables of the form

$$f(x, y) = a_0 y^D + a_1(x) y^{D-1} + \cdots + a_D(x),$$

where D is the degree of f . For $r > 0$, let

$$G^f(r) := \{(x, y) \in \mathbb{R}^2 : |f(x, y)| \leq r\}.$$

For $K \subset \mathbb{R}^2$, let $z(G^f(r) \cap K)$ denote the number of integer points in the set $G^f(r) \cap K$.

We show that if f satisfies the so called *weakly degenerate condition* w.r.t. its Newton polygon $\Gamma(f)$ then there exists a neighborhood Ω_A of the set

$$\left(\{f = 0\} \cup \left\{ \frac{\partial f}{\partial y} = 0 \right\} \right) \cap \{|x| > A\}$$

such that Ω_A is *vertically thin at infinity*, and

$$z(G^f(r) \cap (\mathbb{R}^2 \setminus \Omega_A)) \asymp r^{\frac{1}{d}} \ln^{1-k} r, \text{ as } r \rightarrow \infty,$$

where d is the coordinate of the furthest point in the intersection of the so called complete Newton polygon $\tilde{\Gamma}(f)$ of f and the diagonal, and $k \in \{0, 1\}$ is the dimension of the face of $\tilde{\Gamma}(f)$ containing the point (d, d) in its interior. This is a joint work with Ha Huy Vui.

Kazushi Ueda (University of Tokyo)

Gamma conjecture for Brieskorn-Pham singularities

Abstract: Gamma conjecture of Galkin-Golyshev-Iritani relates the principal asymptotic classes (also known as the central connection matrices) of the quantum cohomologies of Fano manifolds with the combinations of the Gamma classes and the Chern characters of full exceptional collections. In the talk, we will discuss the analogue of gamma conjecture for Brieskorn-Pham singularities, and its relation with homological mirror symmetry.

Adam Parusinski (Université Nice Sophia Antipolis)

Arc-wise analytic equisingularity

Abstract: Using Whitney interpolation, we construct explicit subanalytic trivializations of Zariski equisingular families of complex or real analytic set germs. Moreover, these trivializations are analytic on real analytic arcs and (complex or real) analytic with respect to the parameter. We apply this construction to show Whitney's fibering conjecture and to the general position problem in real and complex algebraic geometry.

Tat Thang Nguyen (Institute of Mathematics, VAST)

Invariance of the global monodromies in families of polynomials on algebraic varieties and Newton polyhedra

Abstract: Let $A \subset \mathbb{C}^n$ be a smooth algebraic set and $f: \mathbb{C}^n \rightarrow \mathbb{C}$ be a polynomial function. There exist finite subsets $B \subset \mathbb{C}$ such that the restriction

$$f: A \setminus f^{-1}(B) \rightarrow \mathbb{C} \setminus B$$

defines a C^∞ -locally trivial fibration. The smallest such set is called the bifurcation set of the restriction map $f|_A$. Assume that the map $f|_A$ is Newton nondegenerate at infinity. The aims of this talk are:

- 1) Construct an explicit finite subset $\Sigma \subset \mathbb{C}$ which contains the bifurcation set of $f|_A$.
- 2) Prove that the global monodromy of the above fibration is completely determined by the Newton polyhedron at infinity of $f|_A$.

This is a joint work with Pham Phu Phat and Pham Tien Son.

Takehiko Yasuda (Osaka University)

Equisingularities and dualities of local field masses

Abstract: I will talk about relation between equisingular families of quotient singularities and dualities of mass formulas of local field extensions. It is the McKay correspondence that relates the two different subjects. The focus will be on families of singularities in positive or mixed characteristics. This talk is based on a joint work with Melanie Wood.

Vincent Grandjean (Universidade Federal do Ceara)

Reduction of regular families of normal operators

Abstract: We would like to present a new proof of results by Kurdyka & Paunescu, Rainer & als, about how to parameterize a locally simultaneous reduction of a regular family of normal operators.

Maria R. Gonzalez-Dorrego (Universidad Autonoma de Madrid)

On singular varieties with smooth subvarieties

Abstract: Let k an algebraically closed field. Let $\text{char } k = 0$. Let Z be a reduced irreducible nonsingular subvariety of a normal n -fold X with certain type of singularities such that $Z \cap \text{Sing}(X) \neq \emptyset$. We study the singularities of X through which Z passes.

Tatsuo Suwa (Hokkaido University)

Relative Dolbeault cohomology and Sato hyperfunctions

Abstract: The Čech-de Rham cohomology is effectively used in various problems related to localization of characteristic classes. Likewise we may develop the Čech-Dolbeault cohomology theory and on the way we naturally come up with the relative Dolbeault cohomology. This cohomology turns out to be canonically isomorphic with the local (relative) cohomology of A. Grothendieck and M. Sato so that it provides a handy way of expressing the latter. In this talk I will present this cohomology theory and give, as applications, simple explicit expressions of the Sato hyperfunctions, some fundamental operations on them and related local duality theorems. The talk includes a joint work with N. Honda and T. Izawa.

Tuan Hiep Dang (National Center for Theoretical Sciences, Taiwan)

The localization technique and its application to Schubert calculus

Abstract: The localization technique in equivariant cohomology is a powerful tool in many different fields, especially algebraic geometry, algebraic topology, symplectic geometry, algebraic combinatorics, and singularity theory. The first goal of this talk is to recall the localization formula in the theory of equivariant cohomology which was proved independently by Atiyah-Bott and Berline-Vergne in 1984. Then I will present an application to Schubert calculus on the Lagrangian Grassmannian. In particular, the Schubert structure constants in this case will be formulated. This is based on arXiv:1612.09177.

Yimu Yin (Los Angeles)

Bounded motivic integral and motivic Milnor Fiber

Abstract: Using the integration theory of Hrushovski and Kazhdan, Hrushovski and Loeser gave a construction of the Denef-Loeser motivic Milnor fiber avoiding the use of resolution of singularities. Unfortunately, there is an oversight in the construction. I will describe the work needed to correct this. If time permits, I will also describe how our method yields a recent tropical motivic Fubini theorem of Nicaise-Payne.

Takuro Abe (Kyushu University)

Hyperplane arrangements and Hessenberg varieties

Abstract: Hessenberg varieties were introduced by De Mari, Procesi and Shayman in 1992 as a generalization of flag varieties and Springer fibers, and recently several connections to geometric representation theory and combinatorics are found, and intensively studied. However, there had been no uniform way to describe their cohomology group and even Betti numbers. In this talk, we give a presentation of their cohomology groups in terms of logarithmic derivation modules of certain hyperplane arrangements, which generalizes Borel's result on the isomorphism between the coinvariant algebra and the cohomology group of flag varieties. This is a joint work with Tatsuya Horiguchi, Mikiya Masuda, Satoshi Murai and Takashi Sato.

Anne Pichon (Aix-Marseille Université)

Lipschitz normal embedding of complex surface singularities

Abstract: Any germ of a complex analytic space $(X, 0) \subset (\mathbb{R}^n, 0)$ is equipped with two metrics: the *outer metric* induced by the euclidean metric of the ambient space and the *inner metric*, which is the associated length metric on the germ. The two metrics are natural in the sense that up to local bilipschitz homeomorphisms, they do not depend on the choice of embedding in some $(\mathbb{R}^n, 0)$. These two metrics are in general nonequivalent up to bilipschitz homeomorphism. We say that $(X, 0)$ is *Lipschitz normally embedded* if it is the case. I will present some recent results about Lipschitz normally embedding of normal surface singularities.

Zhenjian Wang (Université Cote d'Azur)

On deformations of nodal hypersurfaces

Abstract: In this talk, we extend the infinitesimal Torelli theorem from smooth

projective hypersurfaces to nodal hypersurfaces.

Kyoji Saito (IPMU, University of Tokyo)

Period domain of for primitive forms for uni-modular exceptional singularities

Abstract: We give a conjectural description of the flat structure on the ring of automorphic forms on a one dimensionally extended domain of a bounded symmetric domain of type IV. The structure appears in the study of period map associated with primitive forms for the 14 exceptional uni-modular singularities. In physics terminology, the one dimensional extension corresponds to a deformation to an irrelevant direction in the renormalization group action. Mathematically, the extended domain is mirror symmetric to the space of stability conditions on certain K3-surfaces, and the deformation is negatively graded.

The mini-school and symposium will be in Room 101 in the building labeled 18 on this Campus Map.

