Workshop
"The Mathematics of Shapes"

Date: 30 - 31 August 2024

Venue: Room 3-309, Building 3, Faculty of Science, Hokkaido University

Organizers:

Goro Akagi (Tohoku University)
Lorenzo Cavallina (Tohoku University)
Hideo Kubo (Hokkaido University)
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Website:

`https://sites.google.com/view/hprt2024/workshop/`

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Program

30 August 2024

09:00 - 09:15	Opening and welcoming
09:15 - 10:30	Giorgio Poggesi (University of Western Australia) On critical points of some volume-constrained shape optimization problems
10:30 - 10:45	Break
10:45 - 12:00	Kei Funano (Tohoku University) Domain monotonicity, its converse, and reverse
12:00 - 13:30	Lunch
13:30 - 14:45	Shigeru Sakaguchi (Tohoku University) Interaction between initial behavior of temperature and the mean curvature of the interface in two-phase heat conductors
14:45 - 15:00	Break
15:00 - 16:15	Tomoyuki Oka (Fukuoka Institute of Technology) On a two-material optimal design problem with nonlinear boundary conditions
16:15 - 16:30	Break
16:30 - 18:00	Poster session
19:30 -	Social Dinner

31 August 2024

09:30 - 10:45	Ilaria Lucardesi (University of Pisa) Shape optimization under width constraint
10:45 - 11:15	Break
11:15 - 12:30	Tatsuya Miura (Kyoto University) Uniqueness and minimality of Euler's elastica with monotone curvature
12:30 - 14:00	Lunch

14:00 - 15:15	Hirofumi Notsu (Kanazawa University) An adaptive pressure-stabilized Lagrange-Galerkin method for incompressible flow problems
15:15 - 15:30	Break
15:30 - 16:45	Nobuhito Miyake (Kyushu University) Threshold-type algorithm for the gradient flow of Willmore-type functionals
16:45	Closing

Abstracts

On critical points of some volume-constrained shape optimization problems

Giorgio Poggesi (University of Western Australia)

The talk will focus on critical points of some isoperimetric-type problems, or, more precisely, volume-constrained shape optimization problems.

The celebrated Alexandrov's Soap Bubble Theorem and Serrin's symmetry result provide characterizations of critical points for the isoperimetric problem associated with the area functional (the classical isoperimetric problem) and the isoperimetric problem associated with the torsional rigidity functional (Saint Venant's problem), respectively.

We present various generalizations of these celebrated symmetry theorems and discuss related quantitative stability results.

The talk will be based on

[1] G. Poggesi, Soap bubbles and convex cones: optimal quantitative rigidity, Trans. Amer. Math. Soc. (2024). DOI: `https://doi.org/10.1090/tran/9207`

and will also review results obtained in:

[2] L. Cavallina, G. Poggesi, T. Yachimura, Quantitative stability estimates for a two-phase Serrin-type overdetermined problem, Nonlinear Anal. **222** (2022), Paper No. 112919, 17 pp.

[3] G. Ciraolo, S. Dipierro, G. Poggesi, L. Pollastro, E. Valdinoci, Symmetry and quantitative stability for the parallel surface fractional torsion problem, Trans. Amer. Math. Soc. **376** (2023), no. 5, 3515–3540.

[4] S. Dipierro, G. Poggesi, J. Thompson, E. Valdinoci, Quantitative stability for overdetermined nonlocal problems with parallel surfaces and investigation of the stability exponents, J. Math. Pures Appl. (9) **188** (2024), 273–319.

[5] R. Magnanini and G. Poggesi, Serrin's problem and Alexandrov's Soap Bubble Theorem: enhanced stability via integral identities, Indiana Univ. Math. J. **69** (2020), no. 4, 1181–1205.

[6] R. Magnanini and G. Poggesi, Nearly optimal stability for Serrin's problem and the Soap Bubble theorem, Calc. Var. Partial Differential Equations **59** (2020), no. 1, Paper No. 35, 23pp.

[7] F. Pacella, G. Poggesi, A. Roncoroni, Optimal quantitative stability for a Serrin-type problem in convex cones, Math. Z. **307** (2024), no. 4, Paper No. 79.

[8] G. Poggesi, Remarks about the mean value property and some weighted Poincaré-type inequalities, Ann. Mat. Pura Appl. (4) **203** (2024), no. 3, 1443–1461.

[9] S. Dipierro, G. Poggesi, J. Thompson, E. Valdinoci, Quantitative stability for the nonlocal overdetermined Serrin problem, preprint (2023) `arXiv:2308.11203`

[10] G. Poggesi, Bubbling and quantitative stability for Alexandrov's Soap Bubble Theorem with L^1-type deviations, preprint (2024) `arXiv:2405.06376`

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Domain monotonicity, its converse, and reverse

Kei Funano (Tohoku University)

In this talk I will discuss about domain monotonicity for eigenvalues of the Laplacian on bounded convex domains in Euclidean space. I will also discuss its converse statement, and its reverse inequality.

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Interaction between initial behavior of temperature and the mean curvature of the interface in two-phase heat conductors

Shigeru Sakaguchi (Tohoku University)

We consider the Cauchy problem for the heat diffusion equation in the whole Euclidean space consisting of two media with different constant conductivities, where initially one medium has temperature 0 and the other has temperature 1. Suppose that the interface S is of class C^2 in a neighborhood of a point $x \in S$. We extract the mean curvature of S at x from the initial behavior of temperature at x. This result is purely local in space. As a corollary, it is shown that if the interface is of class C^2 and is stationary isothermic, then the mean curvature of the interface must be constant.

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On a two-material optimal design problem with nonlinear boundary conditions

Tomoyuki Oka (Fukuoka Institute of Technology)

In this talk, we consider an optimal design problem for a material consisting of two materials with different diffusion coefficients. Our goal is to determine the two-material domains that minimize the so-called Dirichlet energy, described by the solution to the steady-state diffusion equation with nonlinear boundary conditions. To this end, we discuss the existence of optimal two-material domains using homogenization theory and construct such domains numerically. This talk is based on joint work with Kosuke Kita (Tohoku University) and Kei Matsushima (The University of Tokyo).

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Shape optimization under width constraint

Ilaria Lucardesi (University of Pisa)

Among the geometric constraints considered in shape optimization, the classical ones are surely the perimeter and the volume. For example, the literature on minimizing the eigenvalues of the Laplacian under one of these constraints spans more than a century of Mathematics. Less common, but equally significant, are the constraints of maximal width (the diameter) and minimal width, which describe how 'large' and 'small' an admissible set can be, respectively. The shape optimization of monotone functionals (with respect to inclusion) under maximal/minimal width constraint naturally leads to the consideration of two classes of shapes: the bodies of constant width and the reduced bodies. The classical literature on shape optimization in these familes of sets focuses on functionals of purely geometric type. The coupling with functionals of spectral type is a new research field and has many open questions. In this seminar, I will present some recent results and ongoing projects in this direction. The talk is based on joint works with Antoine Henrot (Nancy, France) and Davide Zucco (Turin, Italy).

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Uniqueness and minimality of Euler's elastica with monotone curvature

Tatsuya Miura (Kyoto University)

For an old problem of Euler's elastica we prove the novel global property that every planar elastica with non-constant monotone curvature is uniquely minimal subject to the clamped boundary condition. We also discuss its extension and application.

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An adaptive pressure-stabilized Lagrange-Galerkin method for incompressible flow problems

Hirofumi Notsu (Kanazawa University)

A stabilized Lagrange-Galerkin method of second-order in time with an adaptive mesh refinement technique for incompressible flow problems is presented. For linear problems, error estimates of second-order in time and first-order in space are proved in the framework of L^2-theory, where the pressure-stabilization technique is employed. As an application, the method is applied to a two-fluid flow problem, and two- and three-dimensional numerical results are shown.

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Threshold-type algorithm for the gradient flow of Willmore-type functionals

Nobuhito Miyake (Kyushu University)

In this talk, we consider the threshold-type algorithm for the gradient flow of Willmore-type functionals by using a higher order parabolic equation. The threshold-type algorithm to the geometric evolution equation was firstly introduced by Bence-Merriman-Osher in 1992. In that study, they used the level set of the solution to the Cauchy problem for the heat equation in order to construct an approximation of the mean curvature flow. In this talk, we define a new threshold function by using a fourth order parabolic equation instead of the heat equation, and construct an approximation for the gradient flow of Willmore-type functionals. This talk is based on a joint work with Professor Katsuyuki Ishii (Kobe University), Professor Yoshihito Kohsaka (Kobe University), and Professor Koya Sakakibara (Kanazawa University).