Cahn-Hilliard systems with nonlinear diffusion

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In this talk we will focus on a class of Cahn-Hilliard models characterized by nonlinear diffusion dynamics. These models, which arise in the physics of polymers and of water-oil-surfactant mixtures, have an independent mathematical interest as they can be viewed as gradient flow problems for the free energy functional

$$E(u) = \int_\Omega \left(\frac{a(u)}{2} |\nabla u|^2 + F(u) \right),$$

where $F(u)$ is the standard logarithmic potential and $a(u)$ is a nonlinear function giving rise to the nonlinear diffusion effect in the equation. We will first consider the case when a is smooth, bounded and strictly positive. Then, we will consider the singular case corresponding to $a(u) \sim (1-u^2)^{-1}$. In both cases we will prove existence of a weak solution. Moreover, we will discuss about uniqueness and parabolic regularization properties, which hold in special cases. The results discussed in this talk have been obtained in collaboration with Irena Pawlow (Military University of Technology and Polish Academy of Sciences, Warsaw).