CONFERENCE ON

Evolution Equations, Related Topics and Applications

Waseda University, Tokyo, Japan

March 19 – 23, 2012

Supported by JSPS (Japan) and DFG (Germany)

Collaborated with Research Institute of Nonlinear PDEs (Waseda University)

Scientific Organizers:

Germany	**Japan**
Messoud Efendiev (München)	Nobuyuki Kenmochi (Kyoto)
Karl-Heinz Hoffmann (München)	Mitsuharu Ôtani (Tokyo)
Angela Stevens (Münster)	Atsushi Yagi (Osaka)

Local Organizers:

Goro Akagi (Kobe University)
Mitsuharu Ôtani (Waseda University)
Tohru Ozawa (Waseda University)
Yoshihiro Shibata (Waseda University)
Kazunaga Tanaka (Waseda University)
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Website: `http://www.otani.phys.waseda.ac.jp/conf/eveq12/`

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Preface

Dear participants,

On behalf of the Organizing and Scientific Committee, let me welcome all the participants joining us for the Japan-Germany bilateral conference on

Evolution Equations, Related Topics and Applications
March 19 – March 23, 2012

held at

Faculty of Science and Engineering,
Waseda University,

which is supported by the Japan Society for the Promotion of Science and Deutsche Forschungsgemeinschaft and is collaborated with Research Institute of Nonlinear PDEs at Waseda University.

This conference is the direct continuation of the first bilateral conference between Japan and Germany on Evolution Equations and Related Topics which was held at Institute of Biomathematics and Biometry, Helmholtz Zentrum München from September 7 to September 11, 2009.

The aim of this conference is to bring together mathematical scientists from both countries with interest in the theory of evolution equations and its applications to the field of various mathematical sciences and to promote a positive and friendly atmosphere around them to bring creative developments in mathematical sciences.

We hope that this conference would play a role as the catalyzer for fruitful reactions for this purpose.

With my best wishes,

Mitsuharu Ôtani

	Monday (19 March)	Tuesday (20 March)	Wednesday (21 March)	Thursday (22 March)	Friday (23 March)
09:00–09:30	Registration				
09:30–10:00	Opening				
10:00–10:50	**Hoffman**	**Yagi**	**Lukacova**	**Ozawa**	**Sonner**
10:50–11:40	**Kenmochi**	**Wohlmuth**	**Suzuki**	**Rohde**	**Aiki**
11:40–12:00	Coffee Break				
12:00–12:50	**Stevens**	**Shibata**	**Lorenz**	**Yamada**	**Knüpfer**
12:50–14:00	Lunch				Closing
14:00–14:50	**Mimura**	**Kostina**	**Kurokiba**	Excursion (Meet on the **ground floor, 55-Bldg** at **13:50**)	
14:50–15:40	**Fuhrmann**	**Ogawa**	**Neukamm**		
15:40–16:10	Coffee Break				
16:10–17:00	**Senba**	**Meunier**	**Ishiwata**		
17:00–17:50	**Röger**	**Akagi**	**Eisenhofer**		
17:50–	Welcome Party	**Poster Session**	Conference Dinner (from 19:00)		

Date and Venue

Date: March 19-23, 2012

Venue: 2nd floor, 55S-Building, Nishi-Waseda Campus, Waseda University,
3-4-1 Okubo, Shinjuku-ku, Tokyo 169-8555, Japan

Program

Monday: 19 March

09:00–09:30 Registration

09:30–10:00 Opening

10:00–10:50 Karl-Heinz Hoffmann (TU München)
Modeling of CO_2 sequestration and numerical simulation

10:50–11:40 Nobuyuki Kenmochi (Bukkyo University)
Weak solvability for parabolic variational inequalities with a class of weakly time-dependent obstacles

11:40–12:00 Coffee Break

12:00–12:50 Angela Stevens (Münster University)
Pattern formation in cell systems with local interaction

12:50–14:00 Lunch

14:00–14:50 Masayasu Mimura (Meiji University)
Modeling of self-organized aggregation: from individuals to populations

14:50–15:40 Jürgen Fuhrmann (Weierstrass Institute for Applied Analysis and Stochastics)
Mathematical and numerical modeling of coupled flow, transport and reactions in electrochemical devices

15:40–16:10 Coffee Break

16:10–17:00 Takashi Senba (Kyushu Institute of Technology)
Stability of stationary solutions to a chemotaxis system in high dimensional spaces

17:00–17:50 Matthias Röger (TU Dortmund)
Confined structures of least bending energy

17:50– Welcome Party

Tuesday: 20 March

10:00–10:50	Atsushi Yagi (Osaka University) **A numerical method for chemotaxis model**
10:50–11:40	Barbara Wohlmuth (TU München) **Saddle point formulations for coupled multi-physics problems**
11:40–12:00	Coffee Break
12:00–12:50	Yoshihiro Shibata (Waseda University) **On the $\mathcal{R}$-sectoriality for the Stokes operator and its application to the Navier-Stokes equations**
12:50–14:00	Lunch
14:00–14:50	Ekaterina A. Kostina (University of Marburg) **Efficient methods for parameter estimation using Krylov-type technique**
14:50–15:40	Takayoshi Ogawa (Tohoku University) **Dissipative estimate for Schrödinger semi-group with a quadratic singular potential**
15:40–16:10	Coffee Break
16:10–17:00	Nicolas Meunier (Paris Descartes University) **Analysis of a non local model for spontaneous cell polarisation**
17:00–17:50	Goro Akagi (Kobe University, Kobe) **A global variational principle for non-equilibrium systems**
17:50–	**Poster Session**

Wednesday: 21 March

10:00–10:50	Maria Lukacova (University of Mainz) **Analysis and simulation of non-Newtonian fluids in moving domains**
10:50–11:40	Takashi Suzuki (Osaka University) **Brownian point vortices - method of the weak scaling limit**
11:40–12:00	Coffee Break
12:00–12:50	Thomas Lorenz (Goethe University Frankfurt) **Differential equations in (possibly) nonlinear spaces: Nonsmooth shapes and stochasticity join**
12:50–14:00	Lunch

14:00–14:50 Masaki Kurokiba (Muroran Institute of Technology)
On a drift-diffusion system of two particles in $\mathbb{R}^2$

14:50–15:40 Stefan Neukamm (Max Planck Institute for Mathematics in the Sciences)
Error estimates in stochastic homogenization

15:40–16:10 Coffee Break

16:10–17:00 Michinori Ishiwata (Fukushima University)
Variational problems associated with Trudinger-Moser type inequalities in unbounded domains

17:00–17:50 Sabine Eisenhofer (Helmholtz Center München)
TBA

19:00– Conference Dinner
at "TOFURO (土風炉)" Japanese Style Restaurant (居酒屋, Izakaya)

Thursday: 22 March

10:00–10:50 Tohru Ozawa (Waseda University)
A sharp bilinear estimate for the Klein-Gordon equation

10:50–11:40 Christian Rohde (University of Stuttgart)
Multi-phase dynamics and non-classical shock waves

11:40–12:00 Coffee Break

12:00–12:50 Yoshio Yamada (Waseda University)
Spreading and vanishing for free boundary problems arising in mathematical biology

12:50–13:50 Lunch

13:50–19:00 Excursion: **Tokyo Tour on "Hato Bus (はとバス) "**
(Meet on the **ground floor, 55-Bldg** at **13:50**)

Friday: 23 March

10:00–10:50 Stefanie Sonner (Helmholtz Center München)
On the well-posedness of a mathematical model of quorum-sensing in patchy biofilm communities

10:50–11:40 Toyohiko Aiki (Gifu University)
Large time behavior of a carbonation front for the one-dimensional concrete carbonation problem with nonlinear Henry's law

11:40–12:00 Coffee Break

12:00–12:50 Hans Knüpfer (University of Bonn)
TBA

12:50– Closing

Title of Poster Presentations

1. Noboru Chikami (Tohoku University)
   Local existence and blow-up criterion for the compressible Navier-Stokes-Poisson equations in Besov spaces

2. Yoichi Enatsu (Waseda University)
   Harmless delays for the global stability of a positive equilibrium of epidemic models

3. Takeshi Fukao (Kyoto University of Education)
   On a variational inequality for a fluid dynamics of Bingham and Navier-Stokes type in three dimension

4. Junichi Harada (Waseda University)
   A single point blow-up for the heat equations with nonlinear boundary conditions

5. Sachiko Ishida (Tokyo University of Science)
   Time global existence of quasilinear degenerate Keller-Segel systems of parabolic-parabolic type

6. Kei Matsuura (Waseda University)
   Nonstationary micropolar flow under some boundary condition

7. Yusuke Murase (Meijo University)
   Existence theorems for nonlinear evolution equations generated by subdifferentials and perturbations

8. Daisuke Naimen (Naruto University of Education)
   Existence of nontrivial solutions for nonlinear Neumann problems with indefinite coefficients

9. Kazuhiro Oeda (Waseda University)
   Effect of a protection zone on a Lotka-Volterra prey-predator model

10. Motohiro Sobajima (Tokyo University of Science)
    Remarks on analyticity for C_0-semigroups generated by elliptic operators in L^p

11. Yuusuke Sugiyama (Tokyo University of Science)
    Global existence of solutions to some quasilinear wave equations in one space dimension

12. Yukino Tomizawa (Chuo University)
    Global existence of unique solutions to nonautonomous differential equations in Banach spaces

13. Kota Uriya (Tohoku University)
    Modified wave operator for the quadratic nonlinear Schrödinger system in two space dimensions

14. Hiroshi Watanabe (Salesian Polytechnic)
    Entropy solutions for strongly degenerate parabolic equations with discontinuous coefficients

15. Noriaki Yamazaki (Kanagawa University)
    A system of elliptic-parabolic variational inequalities with time-dependent constraints
16. Kentarou Yoshii (Tokyo Science University)
    Linear Schrödinger equations with moving Coulomb singularities

Abstracts

Modeling of CO_2 sequestration and numerical simulation

Karl-Heinz Hoffmann
(TU München, München)

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Weak solvability for parabolic variational inequalities with a class of weakly time-dependent obstacles

Nobuyuki Kenmochi
(Bukkyo University, Kyoto)

This is a joint work with T. Fukao (Kyoto University of Education).

Let H be a real Hilbert space and $\varphi(\cdot)$ be a proper, l.s.c., convex and non-negative function on H such that the level sets of φ are compact in H. We denote the effective domain of φ by $D(\varphi)$ and the subdifferential of φ by $\partial\varphi$ in H. For simplicity, we assume that $\varphi(0) = 0$; hence $0 \in \partial\varphi(0)$.

For a fixed positive number T, let $\{K(t)\} := \{K(t)\}_{0\le t\le T}$ be a family of non-empty, closed and convex subsets of H such that $0 \in K(t)$ and $K(t) = \overline{K(t) \cap D(\varphi)}$ for all $t \in [0, T]$; we define $\varphi_{K(t)}(z) := \varphi(z) + I_{K(t)}(z)$ for $z \in H$, $I_{K(t)}(\cdot)$ being the indicator function of $K(t)$. Given $f \in L^2(0, T; H)$ and $u_0 \in K(0)$, we consider an obstacle evolution problem:

$$u'(t) + \partial\varphi_{K(t)}(u(t)) \ni f(t),\ 0 < t < T,\ u(0) = u_0; \tag{1}$$

its weak formulation is

$$\begin{aligned}&\int_0^\sigma (\eta'(\tau), u(\tau) - \eta(\tau))_H d\tau + \int_0^\sigma \varphi(u(\tau))d\tau + \frac{1}{2}|u(\sigma) - \eta(\sigma)|_H^2 \\ &\le \int_0^\sigma \varphi(\eta(\tau))d\tau + \int_0^\sigma (f(\tau), u(\tau) - \eta(\tau))_H d\tau + \frac{1}{2}|u_0 - \eta(0)|_H^2, \\ &\qquad \forall\sigma \in (0, T], \quad \forall\eta \in \mathcal{K}_0,\end{aligned} \tag{2}$$

where $\mathcal{K}_0 := \{\eta \in W^{1,2}(0, T; H); \eta(t) \in K(t),\ \forall t \in [0, T], \varphi(\eta(\cdot)) \in L^1(0, T)\}$.

It is well known that problem (1) has a unique solution $u \in W^{1,2}(0, TH)$, if the set-valued mapping $t \to K(t)$ is "smooth" in a certain sense. In this talk, in case this mapping is continuous in time, but not necessarily "smooth", we discuss the weak solvability of (2) in the space $C([0, T]; H)$.

This work was motivated by the following quasi-variational problems:

(A) Fluid flow with temperature dependent velocity constraint.

(B) Activation of bacteria in temperature dependent environment.

In both cases, the constraint set depends on the unknown temperature and its time-dependence can not be expected to be "smooth", but expected to be continuous.

Pattern formation in cell systems with local interaction

Angela Stevens
(Münster University, Münster)

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Modeling of self-organized aggregation: from individuals to populations

Masayasu Mimura
(Meiji University, Tokyo)

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Mathematical and numerical modeling of coupled flow, transport and reactions in electrochemical devices

Jürgen Fuhrmann
(Weierstrass Institute for Applied Analysis and Stochastics, Berlin)

Electrochemical devices have manifold applications in technical systems. In particular, the growing share of fluctuating rewnewable sources in the supply of energy calls for the ability to store electrical energy in significantly larger quantities as before. In this context, electrochemical storage methods based on secondary batteries, fuel cells, redox flow cells and electrolysis cells are in the focus of significant new research efforts, which include modeling from the molecular scale up to the system scale.

At the macroscale of a single electrochemical cell, coupled nonlinear systems of PDEs describe tightly coupled flow, transport, reactions and electric field. In the talk, we review a elements of mathematical models of this kind, and discuss mathematical and numerical challenges concerning their investigation.

In this context, we discuss the advantages and challenges of the implicit Euler, Voronoi box based finite volume method which allows to derive a framework for the numerical implementation of mathematical models based on reaction-diffusion-convection systems. Particular advantages of the method are unconditional stability, positivity, discrete maximum principle, local and global mass conservation, and efficient ways to solve stationary and time dependent cases. It relies on the ability to create Delaunay meshes conforming to interior and exterior boundaries. We mention challenges in connection with the resolution of boundary layers and handling of anisotropies.

We present results of cooperation with electrochemical groups concerning the modeling of direct methanol fuel cells and thin layer flow cells.

Stability of stationary solutions to a chemotaxis system in high dimensional spaces

Takasi Senba
(Kyushu Institute of Technology, Kita-Kyushu)

We consider classical solutions to a parabolic-elliptic system in high dimensional spaces. The system is a simplified version of a chemotaxis model, and is also a model of self-interacting particles. It is well known that solutions to the system have many kinds of behaviors. If the initial function is sufficiently small in the sense of a suitable functional space, the solution exists globally in time. If the initial function is sufficiently large in the sense of a suitable functional space, the solution blows up in finite time. In addition, there exist solutions blowing up in infinite time. In two dimensional case, it was shown that radial stationary solutions are stable under radial and small perturbations, which means that some radial solutions converge to a stationary solutions. The speaker and Y. Naito constructed oscillating solutions in two dimensional case, by using the stability of these stationary solutions. We will talk about a stability of radial stationary solutions and the existence of oscillating solutions in high dimensional case.

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Confined structures of least bending energy

Matthias Röger
(TU Dortmund, Dortmund)

We consider smooth embeddings of the sphere into the unit ball that satisfy an additional constraint on the surface area. We estimate the minimal Willmore energy in this class and discuss the behavior when the prescribed area just exceeds the area of the unit sphere. (This is joint work with Stefan Müller, Bonn.)

A numerical method for chemotaxis model

Doan Duy Hai and Atsushi Yagi
(Osaka University, Osaka)

We are concerned with a chemotaxis-growth model

$$\begin{cases} \dfrac{\partial u}{\partial t} = a\Delta u - \mu\nabla\cdot[u\nabla\chi(\rho)] + f(u) & \text{in} \quad \Omega\times(0,\infty), \\ \dfrac{\partial \rho}{\partial t} = b\Delta\rho - c\rho + \nu u & \text{in} \quad \Omega\times(0,\infty), \\ \dfrac{\partial u}{\partial n} = \dfrac{\partial \rho}{\partial n} = 0 & \text{on} \quad \partial\Omega\times(0,\infty), \\ u(x,0) = u_0(x), \;\; \rho(x,0) = \rho_0(x) & \text{in} \quad \Omega, \end{cases} \tag{CG}$$

presented by Mimura and Tsujikawa [8] for describing the process of pattern formation by some chemotactic biological individuals. Here, $u(x,t)$ denotes the population density of biological individuals at a position $x \in \Omega \subset \mathbb{R}^2$ and time $t \in [0,\infty)$, respectively, and $\rho(x,t)$ the concentration of chemical substance at $x \in \Omega$ and $t \in [0,\infty)$. Biological individuals have, besides random walking, a tendency to move toward higher concentration of the chemical substance. Such an effect is called chemotaxis, for detail see Budrene-Berg [3] and for mathematical modeling see Keller-Segel [7] and Murray [9]. The chemical substance is produced by the biological individuals themselves. $a > 0$ and $b > 0$ are diffusion rates of u and ρ respectively. $c > 0$ and $d > 0$ are degradation and production rates of ρ respectively. $\chi(\rho)$ is a function of $0 \le \rho < \infty$ representing the chemotactic sensitivity of biological individuals. $f(u)$ is a function of $0 \le u < \infty$ representing the growth rate of u. Ω is a two-dimensional bounded domain.

In this talk we intend to introduce a different approach in solving chemotaxis-growth model numerically from the former ones [1, 2, 4]. The spatial discretization for (CG) generated a huge system of ordinary differential equations [4]. The right hand side of such system consists of two terms with different properties. The discretization term derived from the diffusion and reaction is very stiff that is normally required an implicit L-stable method for temporal integration (see [5]). The chemotaxis term is normally lead to a nonlinear and non-smooth semi-discretization system which is very suitable for an explicit scheme (see [6]). We are then going to deal these different terms in the semi-discretization systems by specified integration methods. The stiff terms is treated by using Rosenbrock methods which guarantees the A-stability. On the other hand, a strong-stability-preserving method is used for the discretization of the advection term.

References

[1] M. Aida, *Global behaviour of solutions and pattern formation for chemotaxis-growth equations* (in Japanese), Doctoral thesis, Osaka University (2003).

[2] M. Aida and A. Yagi, *Target pattern solutions for chemotaxis-growth system*, Math. Sci. Jpn **59**(2004), 577–590.

[3] E.O. Budrene and H.C. Berg, *Complex patterns formed by motile cells of Escherichia coli*, Nature **349**(1991) 630–633.

[4] D.D. Hai and A. Yagi, *Numerical computations and pattern formation for chemotaxis-growth model*, Math. Sci. Jpn **70**(2009), 205–211.

[5] E. Hairer and G. Wanner, *Solving Ordinary Differential Equations II*, Springer, 1991.

[6] J.S. Hesthaven and T. Warburton, *Nodal Discontinuous Galerkin Methods*, Springer, 2008.
[7] E.F. Keller and L.A. Segel, *Initiation of slime mold aggregation viewed as instability*, J. Theor. Biol. **26**(1970), 399–415.
[8] M. Mimura and T. Tsujikawa, *Aggregating pattern dynamics in a chemotaxis model including growth*, Physica A **230**(1996), 499–543.
[9] J.D. Murray, *Mathematical Biology II, Spatial Models and Biomedical Applications*, 3rd ed., Springer, 2003.

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Saddle point formulations for coupled multi-physics problems

Barbara Wohlmuth
(TU München, München)

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On the $\mathcal{R}$-sectoriality for the Stokes operator and its application to the Navier-Stokes equations

Yoshihiro Shibata
(Waseda University, Tokyo)

In this talk, I would like to consider the $\mathcal{R}$-sectoriality for the Stokes operator for the compressible viscous fluid flow in a general domain with non-slip condition. By this, we can show the generation of analytic semigroup and the maximal L_p-L_q regularity at the same time. As applications, we show a local in time unique existence theorem in a general domain and a global in time unique existence theorem in a bounded domain for some initial data close to a constant state for the Navier-Stokes equations that describes the motion of compressible viscous fluid flow. This is a joint work with Yuko Enomoto.

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Efficient methods for parameter estimation using Krylov-type technique

Ekaterina A. Kostina
(University of Marburg, Marburg)

Mathematical models are of great importance for sciences and industry. Beside of providing a scientific insight into processes, the mathematical models are used in process optimization and control. However, the results from simulation and optimization are only reliable if an underlying model describe a given process sufficiently precise. This implies a model validated by experimental data with sufficiently good estimates for model parameters. The development and quantitative validation of complex nonlinear differential equation models is a difficult task that requires the support by numerical methods of parameter estimation and the optimal design of experiments.

The realization of methods in practice shows however, that in order to use the complete potential of nonlinear parameter estimation and nonlinear optimum experimental design, we have to deal with several mathematical challenges. One of the challenges is to develop effective algorithms based on iterative linear algebra methods for models which are described by systems of partial differential equations. The talk focuses on Krylov-type methods for solving parameter estimation problems for PDE models and computing the covariance matrix which is important for error assessment of the resulting parameter estimates and for efficient methods for optimum experimental design.

The talk is based on joint work with H.G. Bock, O. Kostyukova, G. Kriwet, M. Saunders and I. Schierle.

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Dissipative estimate for Schrödinger semi-group with a quadratic singular potential

Takayoshi Ogawa
(Tohoku University, Sendai)

We consider the Cauchy problem of the following heat equation with the inverse square potential. For $n \geq 3$ and $\lambda > 0$,

$$\begin{cases} \partial_t u - \Delta u - \dfrac{\lambda}{|x|^2} u = 0, & t > 0, x \in \mathbb{R}^n, \\ u(0) = u_0, & x \in \mathbb{R}^n. \end{cases} \tag{1}$$

It is known by Baras-Goldstein that there exists no positive weak solution if $\lambda > \lambda_* = \frac{(n-2)^2}{4}$ whilst there exists a weak solution if $\lambda \leq \lambda_*$. This problem is closely related to the Hardy inequality of classic type: For $f \in H^1(\mathbb{R}^n)$, $r = |x|$,

$$\lambda_* \int_{\mathbb{R}^n} \frac{|f(x)|^2}{|x|^2} dx \leq \|\nabla f\|_2^2,$$

where $\lambda_* = \frac{(n-2)^2}{4}$ is the best possible constant. The Hardy inequality immediately implies the L^2 well-posedness of the weak solution to the critical singular heat equation (1). Considering the positiveness for the Schrödinger operator

$$H = -\Delta - \frac{\lambda_*}{|x|^2},$$

the Schrödinger semigroup e^{-tH} generated by the Hamiltonian H is well defined in $L^2(\mathbb{R}^n)$ and it satisfies the semigroup properties in $L^2(\mathbb{R}^n)$. Our first question is if we may obtain some improved estimate of the Hardy type for the whole space case and how it may work with the Cauchy problem with the critical Schrödinger semi-group. We then consider the L^p-L^q version of the dissipative estimate for the Schrödinger semi-group. It turns out that the range of the index p is restricted around $p = 2$ because of the strong singularity of the linear potential both at $x = 0$ and $|x| \to \infty$: For any $\frac{2n}{n+2} < q \leq 2 \leq p < \frac{2n}{n-2}$,

$$\|e^{-tH} u_0\|_p \leq C t^{-\frac{n}{2}(1/q - 1/p)} \|u_0\|_q.$$

This talk is based on a joint work with Norisuke Ioku (Ehime Univ.)

Analysis of a non local model for spontaneous cell polarisation

Nicolas Meunier
(Paris Descartes University, Paris)

In this work, we investigate the dynamics of a non-local model describing spontaneous cell polarisation. It consists in a drift-diffusion equation set in the half-space, with the coupling involving the trace value on the boundary. We characterize the following behaviours in the one-dimensional case: solutions are global if the mass is below the critical mass and they blow-up in finite time above the critical mass. The higher-dimensional case is also discussed. The results are reminiscent of the classical Keller-Segel system in double the dimension. In addition, in the one-dimensional case we prove quantitative convergence results using relative entropy techniques. This work is complemented with a more realistic model that takes into account dynamical exchange of molecular content at the boundary. In the one-dimensional case we prove that blow-up is prevented. Furthermore, density converges towards a non trivial stationary configuration.

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A global variational principle for non-equilibrium systems

Goro Akagi
(Kobe University)

This talk is concerned with a global variational formulation for non-equilibrium systems based on the *Weighted Energy-Dissipation* (WED) functional $I_\varepsilon : L^p(0,T;V) \to (-\infty,\infty]$ given by

$$I_\varepsilon(u) := \int_0^T e^{-t/\varepsilon} \left(\psi(u'(t)) + \frac{1}{\varepsilon}\phi(u(t)) \right) \mathrm{d}t$$

for a trajectory $t \in [0,T] \mapsto u(t) \in V$ in a uniformly convex Banach space V. Here u' is the time derivative, $p \in (1,\infty)$, $\psi, \phi : V \to (-\infty,\infty]$ are convex functionals, and ψ has p-growth.

The WED functional arises as a new tool in order to possibly reformulate dissipative evolution problems in a variational fashion. In particular, minimizers u_ε of the WED functional I_ε taking a given initial value $u_\varepsilon(0) = u_0$ are expected to converge as $\varepsilon \to 0$ to solutions of the generalized gradient system (GGS),

$$\partial\psi(u'(t)) + \partial\phi(u(t)) \ni 0, \quad 0 < t < T, \qquad u(0) = u_0 \tag{2}$$

(here ∂ is the subdifferential). The Cauchy problem (2) expresses a balance between the system of *conservative actions* modeled by the gradient $\partial\phi$ of the *energy* ϕ and that of *dissipative actions* described by the gradient $\partial\psi$ of the *dissipation* ψ. This in particular motivates the terminology *WED* as the energy ϕ and dissipation ψ appear in I_ε along with the parameter $1/\varepsilon$ and the exponentially decaying weight $t \mapsto \exp(-t/\varepsilon)$.

We first give a brief introduction to the WED functional formalism for non-equilibrium systems, and then, we present a variational approach to generalized gradient systems (GGS).

This talk is based on a joint work with Ulisse Stefanelli (IMATI-CNR).

Analysis and simulation of non-Newtonian fluids in moving domains

Maria Lukacova
(University of Mainz, Mainz)

We present our recent results on mathematical modelling and numerical simulation of non-Newtonian flows in compliant two-dimensional domains having applications in hemodynamics. Two models of the shear-thinning non-Newtonian fluids, the power law Carreau model and the logarithmic Yeleswarapu model, will be considered. For the structural model the generalized string equation for radially symmetric tubes will be generalized to stenosed vessels and vessel bifurcations. The arbitrary Lagrangian-Eulerian approach is used in order to take into account moving computational domains. The analytical result for the existence of a weak solution for the shear-thickening power-law fluid is based on the global iteration with respect to the domain deformation, energy estimates, compactness arguments using the semi-continuity in time and the theory of monotone operators. We will also present several numerical experiments for the Carreau and the Yeleswarapu model, comparisons of the non-Newtonian and Newtonian models and the results for hemodynamical wall parameters; the wall shear stress and the oscillatory shear index. Numerical experiments confirm high order accuracy and stability of new fluid-structure interaction methods. The results have been obtained in a cooperation with A. Hundertmark, G. Rusnakova (University of Mainz) and S. Necasova (Academy of Sciences, Prague).

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Brownian point vortices - method of the weak scaling limit

Takashi Suzuki
(Osaka University, Osaka)

We provide a mathematical analysis for the mean field equation on Brownian point vortices introduced by P.-H. Chavanis. First, Onsager's theory uses the vortex system associated with the Hamiltonian

$$\hat{H}_N(x_1,\ldots,x_N) = \frac{1}{2}\sum_i \gamma^2 R(x_j) + \sum_{i<j} \gamma^2 G(x_i,x_j),$$

$x_i \in \Omega,\ i = 1, 2, \cdots, N$ where $\Omega \subset \mathbf{R}^2$ is a simply-connected bounded domain with smooth boundary $\partial\Omega$, $G = G(x, x')$ is the Green's function of $-\Delta$ provided with the Dirichlet boundary condition, $R(x)$ stands for the Robin function, and γ denotes the intensity of vortices. We have *quantized blowup mechanism* and Hamiltonian control concerning the blowup points regarded as a *recursive hierarchy*. Thus a distribution function of the particle density arises in the high-energy limit subject to the Boltzmann-Poisson equation. Then the particle distribution emerged from the mean field limit concentrates on finite points as the renormalized inverse temperature approaches critical values, with their location identical to the critical point of the renormalized Hamiltonian.

Under the long-range interaction, however, the equivalence of thermodynamical relations between statistical ensembles used in Onsager's theory is violated. A kinetic mean field equation on *canonical* ensembles is introduced in this context using the system of stochastic differential equations

$$\frac{dx_i}{dt} = \gamma \nabla_{x_i}^{\perp} \hat{H}_N - \mu\gamma^2 \nabla_{x_i} \hat{H}_N + \sqrt{2\nu} R_i(t), \quad i = 1, 2, \cdots, N$$

where μ stands for the mobility in the theory of Brownian motion, $\nu \geq 0$ denotes the diffusion coefficient describing the viscosity of particles, and $R_i(t)$ is the white noise. Under this setting, the N-body distribution function $P_N(x_1, \cdots, x_N, t)$ is subject to the Fokker-Planck equation which induces a BBGKY-like hierarchy. Then the Euler-Smoluchowski-Poisson equation

$$\frac{\partial \omega}{\partial t} + \nabla^{\perp}\psi \cdot \nabla\omega = \nu\nabla \cdot (\nabla\omega + \beta\gamma\omega\nabla\psi), \quad -\Delta\psi = \omega, \quad \psi|_{\partial\Omega} = 0$$

arises, assuming the factorization property (propagation of chaos). Its similarity to the Smoluchowski-Poisson equation is noticed where the stationary state coincides with the Boltzmann-Poisson equation, and blowup solutions in finite time create collapses, the delta-function singularities with the quantized mass.

Another kinetic model is concerned with the two-species vortices which takes a form of the drift-diffusion model with vorticity terms. Its chemical or physical gradient is hetero-repulsive and homo-aggregative. We have a discrepancy of the blowup times between two species, called non-simultaneous blowup. In the standard drift-diffusion model, on the other hand, such gradient is formed in the opposite direction, that is, hetero-aggregative and homo-repulsive. The other two-species model describes a chemotactic competition between the species. In spite of this profile, any radially symmetric solution exhibits simultaneous blowup, that is, the blowup time of two species is the same, while there is a collapse mass separation which means that only one species creates a delta-function singularity, and the singularity of the other component is locally described by an unbounded L^1-local function. In any case the control of the boundary blowup point is easier if the Poisson part is associated with the Neumann boundary condition. Yet, even interior blowup points are difficult to control in the Euler-Smoluchowski-Poisson equation because of the vorticity terms.

The purpose of the present talk is to describe the method of weak scaling limit to guarantee the non-stationary quantized blowup mechanism. First, we take the Smoluchowski-Poisson equation and show several key points of the argument in accordance with the scaling and monotonicity properties. Applications to the blowup in infinite time are also presented. Then we illustrate our results on these models with multi-components, Dirichlet boundary condition of the Poisson part, and vorticity terms.

References

[1] P.-H. Chavanis, *Two-dimensional Brownian vortices*, Physica A **387** (2008) 6917–6942.

[2] E.E. Espejo, A. Stevens, and T. Suzuki, *Simultaneous blowup and mass separation during collapse in an interacting system of chemotactic species*, Differential and Integral Equations **25** (2012) 251–289.

[3] M. Kurokiba and T. Ogawa, *Finite time blow-up of the solution for a nonlinear parabolic equation of drift-diffusion type*, Differential and Integral Equations **16** (2003) 427–452.

[4] T. Suzuki, *Brownian point vortices and DD-model*, preprint

[5] T. Suzuki, *Free Energy and Self-Interacting Particles*, Birkhäuser, Boston, 2005.

Differential equations in (possibly) nonlinear spaces: Nonsmooth shapes and stochasticity join

Thomas Lorenz
(Goethe University Frankfurt, Frankfurt)

Ordinary differential equations play a central role in science. Their theory is well known to be extended successfully from the finite-dimensional Euclidean space to so-called evolution equations in Banach spaces by means of strongly continuous semigroups. For many applications, however, it is difficult to specify a suitable normed vector space. Shapes in three-dimensional space, for example, do not always have an obvious linear structure if any a priori assumptions about regularity are avoided. Hence from the analytical point of view, they should be regarded just as nonempty compact subsets of the Euclidean space. Supplied with the classical Hausdorff distance, they form a metric space, but popular approaches to set addition (like the Minkowski sum) are not invertible.

In the 1990s, Jean-Pierre Aubin introduced differential equations for compact subsets of the Euclidean space. In this talk the main notions of his approach are summarized first. Then an new extension to random closed sets in the Euclidean space is presented. This joint work with Peter E. Kloeden is the first approach which is not restricted to convex random sets.

These two examples of set evolutions prove to fit in a general formalism called mutational analysis which extends ordinary differential equations beyond vector spaces. Its purpose is to unify diverse types of differential problems in time (such as semilinear evolution equations and parabolic differential equations in noncylindrical domains) so that they can be coupled in systems immediately. This provides new analytical tools for applications in modelling.

********** ********** **********

On a drift-diffusion system of two particles in $\mathbb{R}^2$

Masaki Kurokiba
(Muroran Institute of Technology)

We consider the parabolic-elliptic system describing chemotactic aggregation of two kind of particle. For Nagai model describing one kind of particle, T.Nagai showed that the conjecture of Childress and Percus concerning $n = 2$ is true for radially symmetric solutions, that is, the chemotaxis collapse can occur i f the total cell number on $\Omega \subset \mathbb{R}^2$ is larger than 8π and can not occur in the other case. And T. Senba and T. Suzuki corrected this value to 4π in the genera case. Moreover T.Ogawa and M.Kurokiba showed the existence of blowing-up solutions of the elliptic-parabolic system as the drift-diffusion equations of two particles in whole space $\Omega \subset \mathbb{R}^2$ (2003). In this talk we try to find the construction of solutions like the Nagai model for two kind of particle.

Error estimates in stochastic homogenization

Stefan Neukamm
(Max Planck Institute for Mathematics in the Sciences, Leipzig)

We consider the elliptic equation $-\nabla \cdot (a(x)\nabla u) = f$ on the lattice $\mathbb{Z}^d$ with random coefficients $a(x)$. For stationary and ergodic coefficients $a(x)$, it is known from the theory of stochastic homogenization that the solution operator, i. e. the inverse of the elliptic operator $-\nabla \cdot (a(x)\nabla u)$, behaves on macroscopic scales like the inverse of $-\nabla \cdot a_{\text{hom}}\nabla u$ with homogeneous, deterministic coefficients a_{hom}. The *homogenized coefficients* a_{hom} are given by the homogenization formula $a_{\text{hom}}\xi = \langle a(\xi + \nabla\phi)\rangle$ where ξ is a given direction, $\langle\cdot\rangle$ is the ensemble average (describing the statistics) and ϕ is the solution to the *corrector* problem $-\nabla \cdot (a(x)(\xi + \nabla\phi) = 0$ on $\mathbb{Z}^d$.

In practice the homogenization formula has to be approximated, since (I) the corrector problem can only be solved for a small number of realizations of the ensemble $\langle\cdot\rangle$, and (II) the corrector problem can only be solved in a finite domain of large diameter L. In this talk we present an optimal error estimate for the approximation of a_{hom} by the periodization method for independent and identically distributed coefficients. In particular, we prove that the L^2-error in probability decays (for $L \to \infty$) with the rate of the central limit theorem – although a_{hom} is highly nonlinear as a function of $a(x)$.

Our methods involve spectral gap estimates, estimates on the parabolic Green's function and the semigroup associated to the corrector problem.

This is joint work with Antoine Gloria, INRIA Lille, and Felix Otto, MPI Leipzig.

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Variational problems associated with Trudinger-Moser type inequalities in unbounded domains

Michinori Ishiwata
(Fukushima University, Fukushima)

It is known that the classical Trudinger-Moser inequality in bounded domains has several $\mathbb{R}^2$-versions. In this talk, we discuss the existence and nonexistence of a maximizer for the associated variational problem and try to reveal the difference between the problem defined on bounded domains. Particularly, we obtain the nonexistence of maximizers as well as the existence, which reveals a striking difference from the problem associated with bounded domains. Some recent results on the extension of Trudinger-Moser type inequality in unbounded domains with weight function are discussed.

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Title

Sabine Eisenhofer
(Helmholtz Center München, München)

A sharp bilinear estimate for the Klein-Gordon equation

Tohru Ozawa
(Waseda University, Tokyo)

This talk is based on a recent joint work with Keith M. Rogers, Instituto de Ciencias Matemáticas, Madrid.

We prove a sharp bilinear estimate for the free Klein-Gordon equation in 1 + 1 dimensional space-time.

As an application, we prove new estimates on Fourier restriction onto the hyperbola, where the pullback measure is not supposed to be compactly supported.

********** ********** **********

Multi-phase dynamics and non-classical shock waves

Christian Rohde
(University of Stuttgart)

There are a number of multi-phase processes which lead to the consideration of so-called non-classical shock waves to describe the dynamics of the interface (e.g. phase transition waves in compressible materials or saturation overshoots in porous media flow). By a non-classical shock wave we mean here a discontinuous weak solution such that the Laxian stability condition on the number of in/outgoing characteristics is violated. In the talk we will discuss several multiscale methods which appear to give a good framework for these types of problems. The micro-scale information will only be effective at localized subsets (i.e. the interface) of the domain under consideration. As the macro-scale model we consider hyperbolic systems of conservation laws while the micro-scale model is a (usually quite complex) regularization of the macro-scale system. The new approach will be demonstrated on the physical examples mentioned above.

From the numerical point of view the major issue remains the efficient computation of the micro-scale model which takes most of the overall computational time. We present several techniques to accelerate the computations which rely on new simplified approximations of the micro-scale problems. In turn this poses analytical questions on the well-posedness and asymptotic behaviour of the simplified models.

Spreading and vanishing for free boundary problems arising in mathematical biology

Yoshio Yamada
(Waseda University, Tokyo)

This talk is concerned with the following free boundary problems:

$$
\text{(E)}\begin{cases} u_t = du_{xx} + f(u), & 0 < x < s(t), t > 0, \\ u(0,t) = u(s(t),t) = 0, & t > 0, \\ \dot{s}(t) = -\mu u_x(s(t),t), & t > 0, \\ u(x,0) = u_0(x) \geq 0, & 0 < x < \ell, \\ s(0) = \ell_0 > 0, \end{cases}
$$

where d and μ are positive constants and f is a smooth function satisfying $f(u) < 0$ for $u > K$ with some $K > 0$. The related free boundary problem has been studied by Du and Lin in order to describe the invasion problem of some species into a new environment. We will discuss the asymptotic behavior of solutions of $(u(t), s(t))$ of (E) as $t \to \infty$; in particular, spreading and vanishing properties of $(u(t), s(t))$.

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On the well-posedness of a mathematical model of quorum-sensing in patchy biofilm communities

Stefanie Sonner
(Helmholtz Center München, München)

(joint work with H. Eberl, M.A. Efendiev)

Quorum-sensing is a communication mechanism used by cells to coordinate behaviour in groups. Recently, a model describing the process of quorum-sensing in a growing bacterial biofilm community was introduced. It consists of four density-dependent reaction-diffusion equations and combines a deterministic continuum model for the growth of bacterial biofilms with a model of quorum-sensing in suspended populations.

The dependent variables are the up-regulated and down-regulated biomass, the concentration of the quorum-sensing molecule and the growth-limiting substrate. The equations for the biomass fractions comprise two non-linear diffusion effects, which cause difficulties in the mathematical analysis: Power-law degeneracy (like in the porous medium equation) as the dependent variable vanishes and power law singularity (like in the fast diffusion equation) as the variable approaches its maximum value. We prove the existence and uniqueness of solutions and show numerical simulations to illustrate the model behaviour.

Large time behavior of a carbonation front for the one-dimensional concrete carbonation problem with nonlinear Henry's law

Toyohiko Aiki
(Gifu University, Gifu)

This is a joint work with Professor Adrian Muntean, Technical University of Eindhoven, the Netherlands.

In [2] Böhm and Muntean have proposed a one-dimensional free boundary problem as a mathematical model for concrete carbonation. Recently, we study the simplified model of their's one (see [1]). In our model Henry's law is described by the linear function. The aim of this talk is to deal with the free boundary problem with nonlinear Henry's law as follows. First, we assume that the carbonated region $Q_s(T)$ is given as $Q_s(T) := \{(t,x) : 0 < x < s(t), 0 < t < T\}$, where $x = s(t)$ indicates the depth of carbonation. Let u and v be concentrations of carbon dioxide in water and air regions, respectively. We suppose that u and v satisfy

$$\begin{aligned}
&u_t - (\kappa_1 u_x)_x = f(u,v), v_t - (\kappa_2 v_x)_x = -f(u,v) \quad \text{in } Q_s(T),\\
&u(t,0) = g(t), v(t,0) = h(t) \quad \text{for } 0 \le t \le T,\\
&s'(t) = \alpha\psi(u(t,s(t))) \quad \text{for } 0 < t < T,\\
&-\kappa_1 u_x(t,s(t)) = \psi(u(t,s(t))) + s'(t)u(t,s(t)) \quad \text{for } 0 < t < T,\\
&-\kappa_2 v_x(t,s(t)) = s'(t)v(t,s(t)) \quad \text{for } 0 < t < T,\\
&s(0) = s_0 \text{ and } u(0,x) = u_0, v(0,x) = v_0 \quad \text{for } 0 < x < s_0,
\end{aligned}$$

where $f(u,v) = \phi(\gamma v - u)$, ϕ is a increasing and continuous function on R, γ is a positive constant, κ_1 and κ_2 are diffusion coefficients, g and h are given boundary data, $\psi(r) = |[r]^+|^p$, $p > 1$ and α are given constants, and s_0, u_0 and v_0 are initial values of s, u and v, respectively. Now, we denote by P=P(s_0, g, h) the above system.

Our main result is:

Theorem. If $f(u,v) = \phi(\gamma v - u)$ and ϕ is a continuous and increasing function on R with $\phi(0) = 0$ and $\phi(r)r \ge C_\phi |r|^{1+q}$ for$r \in R$, where $q \ge 1$ and C_ϕ is a positive constant, $g, h \in W^{1,2}_{loc}([0,\infty)) \cap L^\infty(0,\infty)$, $0 < g_0 \le g \le g^*$, $0 \le h \le h^*$, $g - g_*, h - h_* \in W^{1,1}(0,\infty)$, where g_* and h_* are positive constants with $\gamma h_* = g_*$, and $u_0, v_0 \in L^\infty(0, s_0)$, $u_0, v_0 \ge 0$ on $(0, s_0)$ and $s_0 > 0$, then the problem P(s_0, g, h) has a weak solution $\{s, u, v\}$ on $[0,\infty)$. Moreover, there exist two positive constants c_* and C_* such that

$$c_*\sqrt{t} \le s(t) \le C_*\sqrt{t+1} \quad \text{for } t \ge 0.$$

References

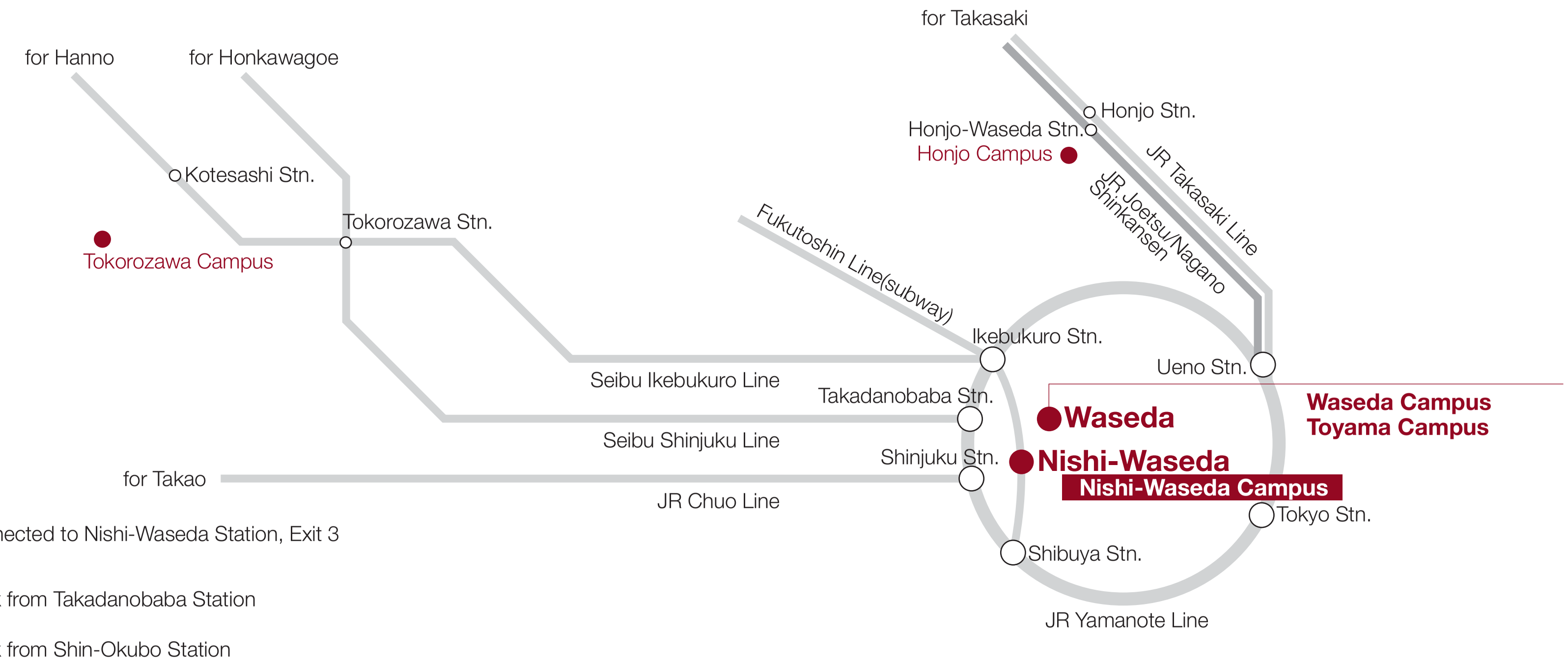
[1] T. Aiki, A. Muntean, Existence and uniqueness of solutions to a mathematical model predicting service life of concrete structures, Adv. Math. Sci. Appl., 19(2009), 109–129.

[2] A. Muntean, M. Böhm, A moving-boundary problem for concrete carbonation: global existence and uniqueness of solutions, Journal of Mathematical Analysis and Applications, 359(1)(2009), 234–251.

Title

Hans Knüpfer (University of Bonn, Bonn)

Transport Map



Fukutoshin Line (subway) — Directly connected to Nishi-Waseda Station, Exit 3

JR Yamanote Line
Tozai Line (subway)
Seibu Shinjuku Line
— 15 min. walk from Takadanobaba Station

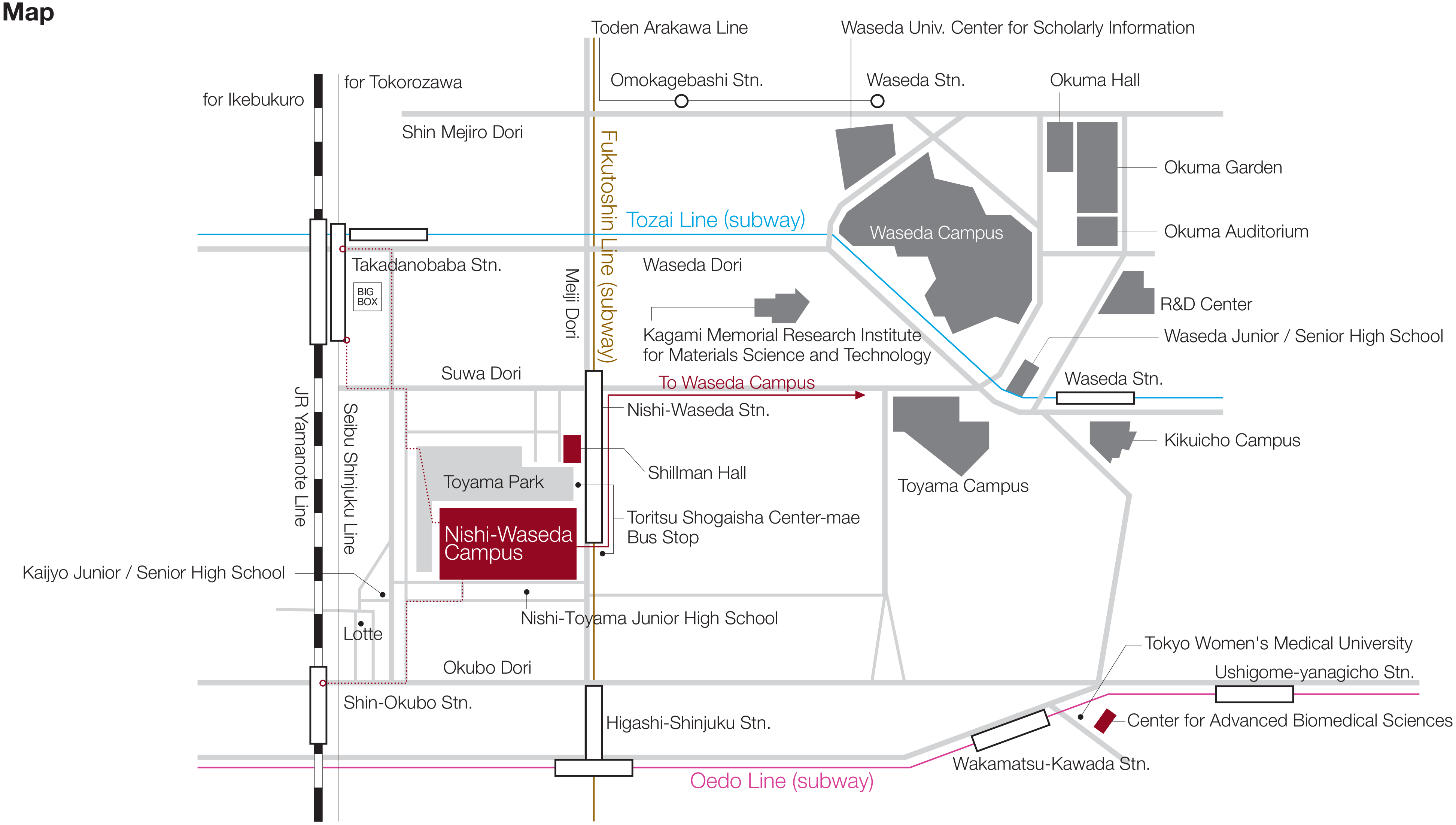
JR Yamanote Line — 12 min. walk from Shin-Okubo Station

Oedo Line (subway) — 15 min. walk from Higashi-Shinjuku Station

City bus
(Ike 86) From Ikebukuro Station (East Exit) to Shibuya Station (East Exit)
(Haya 77) From Shinjuku Station (West Exit) to Waseda
(Taka 71) From Takadanobaba Station to Kudanshita
— 3 min. walk from Toritsu Shogaisha Center-mae bus stop

※Take City Bus (Taka 71) to Tokyo Joshi Idai-mae bus Stop.

Vicinity Map



Nishi-Waseda Campus*

*The name of this campus changed from Okubo Campus to Nishi-Waseda Campus, in April 2009.