

数学特別講義E
確率過程論特論
多様体論特殊講義F III

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談話会

5月12日(月) 16 : 00～

Anderson Hamiltonian and random matrices.

In the 50s, Wigner proposed his famous random matrix model to describe the repulsion observed between energy levels of heavy nuclei. Around the same time, Anderson independently proved a localization phenomenon for electrons, now named “Anderson localization” for some Schrödinger operators. This talk will mention some of the major conjectures of those two fields and explain certain recent results exploiting the links between the two theories.

5月13日(火)～5月16日(金)

各日 15:00～18:00

Localization and delocalization of random matrices and random Schrödinger operators.

Since the 50s, there has been a parallel development of random operator theory and random matrix theory, both initially driven by distinct objectives:

(1) In random matrix theory: One investigates the eigenvalues of large matrices with symmetry properties such as Hermitian, real symmetric, or unitary matrices. The primary questions concern the global and local properties of these eigenvalues. The introduction of those models is due to Eugene Wigner whose motivation was coming from nuclear physics.

(2) In random operator theory: The main object of study concerns the eigenvectors or generalized eigenvectors, and their localization properties i.e. ; how the underlying probability measure associated to the normalized eigenvectors is distributed. A major model in this field is the Anderson model or random Schrödinger operator: $-\Delta + V$ where V is a random potential that models the disorder. Since the breakthrough work of Philip Warren Anderson, intense research studies have been dedicated to investigating this model. The point spectrum of the Schrödinger operator with random weakly correlated potential is fairly well understood by now. The absolutely continuous component remains a mystery. The most important conjecture of this field is to establish the existence of some absolutely continuous spectrum of those operators.

We will study in this course the two theories, and explain what they can gain from each other. On one hand, we will use tridiagonalization of random matrices to get random operators as their scaling limit and therefore understand in deep details their eigenvalue distribution. On the other hand, the eigenvalues of many natural operators are conjectured to correspond to the eigenvalues of famous random matrices.