Eighth Euro-Japanese Workshop on Blow-up

Abstract

June 4-8, 2018

Tohoku University, Sendai, Japan

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ABSTRACTS

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Some classification and Liouville-type theorems for the Fujita equation

Peter Polacik
University of Minnesota

Abstract. In this lecture, entire solutions of the Fujita equation on R^N, both in the subcritical and supercritical cases, will be considered. Several classification and Liouville theorems for such solutions will be presented, and some applications and ideas of the proofs will be discussed.

The "wave-like" blow-up for nonlinear wave equations with the scattering damping [1]

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We are interested in the critical exponent $p_c(n)$ of the following problem.

$$\begin{cases} u_{tt} - \Delta u + \dfrac{\mu}{(1+t)^\beta} u_t = |u|^p \quad \text{in} \quad \mathbf{R}^n \times [0,\infty), \\ u(x,0) = \varepsilon f(x), u_t(x,0) = \varepsilon g(x), \end{cases} \tag{1}$$

where $\mu > 0$, $\beta \in \mathbf{R}$. $\varepsilon > 0$ is a "small" parameter. For simplicity, we assume that $f \in H^1(\mathbf{R}^n)$, $g \in L^2(\mathbf{R}^n)$ with compact support and $n \geq 2$. The "critical" means that, for the energy solution, we have the global-in-time existence when $p_c(n) < p$ and the blow-up in finite time when $1 < p \leq p_c(n)$.

It is known that there is no such a $p_c(n)$ when $\beta < -1$ (overdamping case), namely we have the global existence for any $p > 1$. For $-1 \leq \beta < 1$ (effective damping), we know that $p_c(n) = p_F(n)$, where

$$p_F(n) := 1 + \frac{2}{n}$$

is so-called Fujita exponent which is the critical exponent for $u_t - \Delta u = u^p$. For $\beta = 1$ (scaling invariant damping), there is a conjecture such that

$$\begin{cases} \mu \geq \mu_0(n) & \Longrightarrow \quad p_c(n) = p_F(n) \quad \text{(heat-like)}, \\ 0 < \mu < \mu_0(n) & \Longrightarrow \quad p_c(n) = p_S(n+\mu) \quad \text{(wave-like)}, \end{cases} \tag{2}$$

where

$$\mu_0(n) := \frac{n^2+n+2}{n+2} \quad \text{and} \quad p_S(n) := \frac{n+1+\sqrt{n^2+10n-7}}{2(n-1)}$$

is so-called Strauss exponent which is the critical exponent for $u_{tt} - \Delta u = |u|^p$. We note that

$$p_F(n) < p_S(n) \quad \text{and that} \quad 0 < \mu < \mu_0(n) \Longleftrightarrow p_F(n) < p_S(n+\mu).$$

[1] This talk is presented in 8th Euro-Japanese Workshop on Blow-up, Tohoku University, Sendai, Japan, June 4-8, 2018. All the results in this talk are based on joint works with Ning-An Lai (Lishui Univ. China), which are partially supported by the Grant-in-Aid for Scientific Research (C)(No.15K04964), Japan Society for the Promotion of Science.

The existence part of the conjecture (2) is still open. All the references appear in Introduction of [1].

In this talk, I will introduce a new conjecture for $\beta > 1$ (scattering damping) that

$$p_c(n) = p_S(n) \quad \text{for all} \quad \mu > 0. \tag{3}$$

Introducing a multiplier $\exp\left(\mu(1+t)^{1-\beta}/(1-\beta)\right)$ for a differential equation or inequality of

$$\frac{d}{dt}\int_{\mathbf{R}^n} u(x,t)dx,$$

we succeed to prove the subcritical part of (3). Also the lifespan estimate will be discussed. All the results introduced here are from [1]. This multiplier is useful for another nonlinearity $|u_t|^p$ (see [2]) and their combined nonlinearity $|u_t|^p + |u|^q$ (see [3]).

References

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[2] N.-A.Lai and H.Takamura, *Nonexistence of global solutions of nonlinear wave equations with weak time-dependent damping related to Glassey conjecture*, arXiv:1711.07591.

[3] N.-A.Lai and H.Takamura, *Nonexistence of global solutions of wave equations with weak time-dependent damping and combined nonlinearity*, arXiv:1802.10273.

Construction of refined type II blow-up solutions of the Fujita equation and its applications

Yukihiro Seki
Kyushu University

Abstract. The type II blow-up solutions constructed by Herrero and Vel'azquez (1994) play an important role in the study of the Fujita equation. The way of construction has become one of the standard methods to prove the existence of type II blow-up solutions with exact blow-up rates for some class of nonlinear parabolic problems.

In this talk, I will revisit and refine the way of construction. Applications of the refined construction are presented as well.

Existence, uniqueness and blow-up in semilinear heat equations

Robert Laister
UWE Bristol

Abstract. I will present some results for semilinear heat equations of source type having minimal structural conditions on the nonlinearity f. We re-visit the Fujita-type results from the 70s for classical solutions on the whole space and report on some recent progress for f *not* possessing a certain sub-homogeneity condition inherent in earlier works (such as those with f of power law type). As an example, for a certain class of f (such as those considered by Baras & Cohen for complete blow-up) we will see that blow-up of all positive solutions of the PDE is *equivalent* to the blow-up of solutions of an associated scalar ODE.

Also presented are some sharp results on local existence and uniqueness of classical solutions having L^1 initial data, for which our blow-up results are then applicable.

This is joint work with Mikolaj Sierzega and James C. Robinson.

Asymptotic behavior of solutions to the logarithmic diffusion equation

Masahiko Shimojo
Okayama University of Science

Abstract. We investigate the behavior of positive solutions to the Cauchy problem of logarithmic diffusion equation with non-symmetric flux boundary condition at space infinity. We show that extinction of the solution occurs in a finite time and a re-scaled solution converges to the traveling wave. Our results also include some log-concavity properties of solutions. This talk is a joint work with Eiji Yanagida and Peter Takac

Intersection number and applications for semilinear elliptic equations with general supercritical growth

Yasuhito Miyamoto
University of Tokyo

Abstract.

We study radial solutions of the semilinear elliptic equation $\Delta u + f(u) = 0$ under rather general growth conditions on f. We construct a radial singular solution and study the intersection number between the singular solution and a regular solution. Applications to elliptic problems and corresponding parabolic problems are given. To this end, we derive a certain limit equation from the original equation at infinity, using a generalized similarity transformation. Through a certain interesting change of variables, all the limit equations can be reduced into two typical cases, i.e., $\Delta u + u^p = 0$ and $\Delta u + e^u = 0$.

On a nonlocal degenerate parabolic equation

Johannes Lankeit
Universität Paderborn

Abstract. In this talk we will consider a degenerate parabolic equation with nonlocal gradient nonlinearity, which arises in Game Theory. We will deal with the blow-up of large-mass solutions in bounded domains and with the large time behaviour of unit mass solutions also in R^n. In the latter setting we will in particular investigate how the large time behaviour depends on the spatial decay of the initial data. In parts this talk will be based on a joint work with N. Kavallaris and M. Winkler and on another joint work with M. Winkler.

Convergence of solutions for fractional Cahn-Hilliard systems

Goro Akagi
Tohoku University

Abstract. This talk is concerned with the Cauchy-Dirichlet problem for fractional Cahn-Hilliard equations. We shall discuss global (in time) existence of weak solutions, characterization of parabolic smoothing effects (implying under proper condition eventual boundedness of trajectories), and convergence of each solution to a (single) equilibrium. In particular, to prove the convergence result, a variant of the so-called Łojasiewicz-Simon inequality is provided for the fractional Dirichlet Laplacian and (possibly) non-analytic (but C^1) nonlinearities. This talk is based on a joint work with G. Schimperna and A. Segatti (Pavia, IT).

The large diffusion limit for the heat equation with a dynamical boundary condition

Tatsuki Kawakami

Department of Applied Mathematics and Informatics, Ryukoku University, Japan

We consider the problem

$$(P) \qquad \begin{cases} \varepsilon\partial_t u - \Delta u = 0, & x \in \mathbb{R}^N_+, \ \ t > 0, \\ \partial_t u + \partial_\nu u = 0, & x \in \partial\mathbb{R}^N_+, \ \ t > 0, \\ u(x,0) = \varphi(x), & x \in \mathbb{R}^N_+, \\ u(x,0) = \varphi_b(x'), & x = (x',0) \in \partial\mathbb{R}^N_+, \end{cases}$$

where $N \geq 2$, $\mathbb{R}^N_+ := \mathbb{R}^{N-1} \times \mathbb{R}_+$, Δ is the N-dimensional Laplacian (in x), $\partial_t := \partial/\partial t$, $\partial_\nu := -\partial/\partial x_N$, $\varepsilon \in (0,1)$ and φ and φ_b are measurable functions in $\mathbb{R}^N_+$ and $\mathbb{R}^{N-1}$, respectively. For the case where $\varepsilon = 0$, namely the Laplace equation with the homogeneous dynamical boundary condition

$$(L) \qquad \begin{cases} \Delta u = 0, & x \in \mathbb{R}^N_+, \ \ t > 0, \\ \partial_t u + \partial_\nu u = 0, & x \in \partial\mathbb{R}^N_+, \ \ t > 0, \\ u(x,0) = \varphi_b(x'), & x = (x',0) \in \partial\mathbb{R}^N_+, \end{cases}$$

it is well known that the problem (L) possesses a unique global-in-time solution for any measurable initial data. In this talk we construct a global-in-time solution u_ε of the problem (P) and show that, as $\varepsilon \to 0$, it holds that $u_\varepsilon \to u$ (in a suitable sense), where u is the solution of the problem (L).

This is based on a joint works with M. Fila and K. Ishige.

Touchdown behavior for the MEMS problem with variable dielectric permittivity.

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Abstract. We consider a well-known model for micro-electromechanical systems (MEMS) with variable dielectric permittivity, based on the following parabolic equation with singular nonlinearity

$$\begin{cases} u_t - \Delta u = f(x)(1-u)^{-p}, & x \in \Omega, \quad t > 0, \\ u = 0, & x \in \partial\Omega, \quad t > 0, \\ u(0,x) = 0, & x \in \Omega, \end{cases} \tag{4}$$

where Ω is a smooth bounded domain in R^n, $n \geq 1$, $p > 0$ and $f \geq 0$ is a Hölder continuous function.

We study the eventually apparition of singularities in the nonlinear part of the equation. This phenomenon is known as quenching or touchdown. Recently, the question whether or not touchdown can occur at zero points of the permittivity profile f, which had long remained open, was answered negatively in [3] for the case of interior points.

Our aim in [1] is to go further by considering the same question at points of positive but small permittivity. As one of our main results, in any space dimension, we show that touchdown cannot occur at an interior point where the permittivity profile is suitably small. We also obtain a similar result in the boundary case, under a smallness assumption of f in a neighborhood of the boundary.

Moreover, in view of practical considerations of MEMS design, in [2] we give special care to express our smallness conditions in a *quantitative way*, especially in one space dimension, where analytic computations can be made more precise.

We also obtain another kind of results regarding further properties of the touchdown set, some of them locating it far away from the maximum points of f, which confirm the necessity of some kind of smallness condition on f if one wants to prevent touchdown in certain regions of the domain.

References

[1] C. Esteve, Ph. Souplet, No touchdown at points of small permittivity and nontrivial touchdown sets for the MEMS problem, *To appear in Advances in Differential Equations.*

[2] C. Esteve, Ph. Souplet, Quantitative touchdown localization for the MEMS problem with variable dielectric permittivity, *arXiv:1710.04035*

[3] J.-S. Guo, Ph. Souplet, No touchdown at zero points of the permittivity profile for the MEMS problem, *SIAM J. Math. Analysis* 47 (2015), 614–625.

Solutions with time-dependent singular sets for the heat equation with absorption

Jin Takahashi
Tokyo Institute of Technology

Abstract. We consider the heat equation with a superlinear absorption term $\partial_t u - \Delta u = -u^p$ in $\mathbb{R}^n$ and study the existence and nonexistence of nonnegative solutions with an m-dimensional time-dependent singular set, where $n-m \geq 3$. First, we prove that if $p \geq (n-m)/(n-m-2)$, then there is no singular solution. We next prove that, if $1 < p < (n-m)/(n-m-2)$, then there are two types of singular solution. This talk is based on the results of T. and Yamamoto (preprint, arXiv:1712.06065). This is a joint work with Professor Hikaru Yamamoto (Tokyo University of Science).

On Type I singularities in semilinear parabolic problems

Van Tien Nguyen
NYU Abu Dhabi

Abstract. I will talk about the construction and stability of Type I blowup solutions satisfying some prescribed behavior for the two models:

1. The non-variational semmilinear parabolic system

$$\begin{cases} \partial_t u = \Delta u + v|v|^{p-1}, \\ \partial_t v = \mu \Delta v + u|u|^{q-1}, \end{cases} \quad \mu > 0, \ \ p, q > 1, \quad \text{in } \mathbb{R}^N.$$

2. The higher order semmilinear parabolic equation

$$\partial_t u = -(-\Delta)^m u + u|u|^{p-1}, \quad m \in \mathbb{N}^*, \ \ p > 1, \quad \text{in } \mathbb{R}^N.$$

I will focus on explaining two points:

- Formal derivation of (stable) blowup profiles via spectral analysis.
- A two-step procedure on the construction of solutions verifying such blowup profiles.

The results presented in the talk can be found in [Ghoul et al.(2018a)Ghoul, Nguyen, and Zaag] and [Ghoul et al.(2018b)Ghoul, Nguyen, and Zaag].

References

[Ghoul et al.(2018a)Ghoul, Nguyen, and Zaag] T. Ghoul, V. T Nguyen, and H. Zaag. Construction of type I blowup solutions for a higher order semilinear parabolic equation. *arXiv:1805.06616*, 2018a. URL `https://arxiv.org/abs/1805.06616`.

[Ghoul et al.(2018b)Ghoul, Nguyen, and Zaag] T. Ghoul, V. T. Nguyen, and H. Zaag. Construction and stability of blowup solutions for a non-variational parabolic system. *Ann. Inst. H. Poincaré Anal. Non Linéaire*, (to appear), 2018b. URL `https://arxiv.org/abs/1610.09883`.

Blowup criteria for the parabolic-elliptic model of chemotaxis

Grzegorz Karch
Mathematical Institute, University of Wrocław, Poland

Abstract.

Recent results from the references stated below on the parabolic-elliptic Keller-Segel model of chemotaxis in the whole space will be presented. Here, global-in-time solutions are constructed under (nearly) optimal assumptions on the size of radial initial data. Moreover, criteria for blowup of solutions in terms of their local concentrations will be derived.

References

- Piotr Biler, Grzegorz Karch, & Jacek Zienkiewicz, *Optimal criteria for blowup of radial and N-symmetric solutions of chemotaxis systems,* Nonlinearity **28** (12) (2015), 4369–4387.
- Piotr Biler, Tomasz Cieślak, Grzegorz Karch & Jacek Zienkiewicz, *Local criteria for blowup in two-dimensional chemotaxis models,* Discrete Contin. Dyn. Syst. **37** (4) (2017), 1841–1856.
- Piotr Biler, Grzegorz Karch & Jacek Zienkiewicz, *Large global-in-time solutions to a nonlocal model of chemotaxis,* Advances in Math. (2018), to appear. arXiv:1705.03310.
- Piotr Biler, Grzegorz Karch & Dominika Pilarczyk, *Global and blowing up radial solutions in classical Keller–Segel model of chemotaxis,* (2018), in preparation.

Behavior of solutions to an indirect chemotaxis system

Takashi Senba
Fukuoka University

Abstract. In this talk, we will consider solutions to a parabolic system related to Keller-Segel system. The system has blowup solutions and bounded solutions. In particular, our system has a critical number in four dimensional case. In this talk, we will describe behavior of solutions to our system and a difference and the common point of our system and Keller-Segel system.

Existence of self-similar solutions of the nonlinear heat equation with large initial positive data

Flávio Dickstein

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Abstract. We consider the Cauchy problem in $\mathbb{R}^N$ related to the nonlinear heat equation $u_t - \Delta u = |u|^\alpha u$ on $\mathbb{R}^N$, where $\alpha > 0$ and $N \geq 1$. For $0 < \alpha < \frac{4}{N-2}$ and for the initial value $u(0,x) = \mu|x|^{-\frac{2}{\alpha}}$, where $\mu > 0$, we show the existence of infinitely many sign-changing, self-similar solutions. In particular, we construct sign-changing self-similar solutions for positive initial values for which it is known that there does not exist any local, nonnegative solution. Extensions of this result for more general initial data and for the Dirichlet problem in bounded domains will be presented.

Global dynamics of the conformal focusing wave equation

Kenji Nakanishi
Kyoto University

Abstract. We consider classification of global behavior of solutions to the nonlinear wave equation with the focusing cubic power in three space dimensions. Using its conformal invariance, we can characterize the solution sets for scattering and blow-up in terms of blow-up points in a causal diamond after the conformal inversion, and the threshold between the two sets is the boundary of each set. Considering solutions spatially constant in the diamond, existence of all 9 combinations of the forward and the backward behavior is observed, as well as codimension-1 separation around those special solutions. A notable difference from the massive equation around its ground states is the intersection of the threshold sets with the opposite signs.

Blow-up of sign-changing solutions for a one-dimensional semilinear parabolic equation

Eiji Yanagida
Tokyo Institute of Technology

Abstract. This talk is concerned with a nonlinear parabolic equation on a bounded interval with the homogeneous Dirichlet or Neumann boundary condition. Under rather general conditions on the nonlinearity, we consider the blow-up and global existence of sign-changing solutions. It is shown that there exists a nonnegative integer k such that the solution blows up in finite time if the initial value changes its sign at most k times, whereas there exists a stationary solution with more than k zeros. The proof is based on an intersection number argument combined with a topological method.

Reaction-diffusion systems with dissipation of mass: old and new results

Philippe Souplet
LAGA, Université Paris 13 & CNRS

Abstract.

We consider positivity-preserving reaction-diffusion systems of the form

$$\partial_t u_i - d_i \Delta d_i = f_i(u), \qquad u = (u_1, \dots, u_m),$$

under the Neumann boundary conditions, with the structure condition $\sum f_i \leq 0$, which guarantees that the total mass is nonincreasing in time. Such systems are often encountered in applications, for instance in models of reversible chemistry.

Whereas global existence and boundedness of solutions is easy in the equidiffusive case $d_i \equiv d$, the question becomes quite involved in the case when the $d_i > 0$ are different (a case which is indeed relevant in models of chemical reactions), and there has been an abundant mathematical literature on this question in the past 30 years.

Various sufficient conditions on the nonlinearities f_i for global existence are known, as well as examples of finite time blow-up for certain systems. The latter is a special case of the so-called diffusion induced blow-up phenomenon.

We will discuss old and new results on this subject.

On the bounds of the Sobolev norm for solutions of semilinear parabolic problems with critical Sobolev exponent

Michinori Ishiwata
Osaka University

Abstract. In this talk, we summarize the known facts for the bounds of the Sobolev norm for solutions of semilinear parabolic problems with critical Sobolev exponent and try to apply the idea of profile-decomposition to obtain some information about the asymptotics of the Sobolev norm of solutions.

New Lyapunov-like functional of 1D quasilinear Keller-Segel system and its application

Kentarou Fujie[2]
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Abstract. We deal with the fully parabolic 1d chemotaxis system with diffusion $1/(1+u)$. We prove that the above mentioned nonlinearity, despite being a natural candidate, is not critical. It means that for such a diffusion any initial condition, independently on the magnitude of mass, generates global-in-time solution. In order to prove our theorem we establish a new Lyapunov-like functional associated to the system. Moreover we will discuss blowup of solutions in the supercritical case. This talk is based on recent works [1, 2, 3].

References

[1] T. Cieślak, K. Fujie, *No critical nonlinear diffusion in 1D quasilinear fully parabolic chemotaxis system.* Proc. Amer. Math. Soc., **146**, 2529–2540 (2018).

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Pohozaev type identities through special variations

Mikolaj Sierzega
University of Warsaw

Abstract. On the whole space the subcrtitical Fujita equation may only develop singularities with flat local profiles. To show this Giga and Kohn employed a rather non-obvious combination of integral identities. In my talk I'll discuss how their Pohozaev-type identity may be described from the variational standpoint.

Profile of blowup solutions to the heat equation with nonlinear boundary condition

Junichi Harada
Akita University

Abstract. We study the asymptotic behavior of solutions of the heat equation with nonlinear boundary condition. By the presence of a reaction term on the boundary, solutions of this problem can blow up on the boundary. A goal of this talk is to derive the precise boundary behavior of blowup solutions. The proof is based on the matched asymptotic expansion technique as in Herrero-Velazquez's papers. A key part is a study of some limiting equation.

Existence of a multi-soliton solution for the critical nonlinear heat equation

Hiroshi Matano
Meiji University

Abstract. In this talk, I will consider radially symmetric solutions of the nonlinear heat equation

$$u_t = \Delta u + |u|^{p-1}u$$

on $\mathbb{R}^N$, with the Sobolev critical exponent $p = \frac{N+2}{N-2}$. The initial data is taken from the energy space $\dot{H}^1(\mathbb{R}^N)$. I will show, under the assumption $N \geq 7$, the existence of a multi-soliton solution, by which I mean a time-global solution whose large-time profile is given in the form of a superposition of multiple ground states in different orders of scaling.

This is joint work with Frank Merle.

A Gagliardo-Nirenberg-type inequality and its applications to decay estimates for solutions of a degenerate parabolic equation

Marek Fila
Comenius University

Abstract. We discuss a Gagliardo-Nirenberg-type inequality for functions with fast decay. We use this inequality to derive upper bounds for the decay rates of solutions of a degenerate parabolic equation. Moreover, we show that these upper bounds, hence also the Gagliardo-Nirenberg-type inequality, are sharp in an appropriate sense. This is a joint work with Michael Winkler.