

# Mini-workshop on Geometry

**Date:** 10:00–12:00/13:30–17:20, November 26th (Thursday), 2015

**Place:** The building of mathematics: Room 201 (before noon), Room 107(after midday)

**Organizers:** Shin-ichi Matsumura, Reiko Miyaoka (contact [r-miyaok@m.tohoku.ac.jp](mailto:r-miyaok@m.tohoku.ac.jp))

## Program

- 10:00 - 10:40 Norihiko Suzuki  
Stabilities of rough curvature-dimension condition
- 10:50 - 11:50 Masato Mimura  
Strong algebraization of fixed point property
- 11:50 - 13:30 LUNCH
- 13:30 - 14:10 Takashi Komatsu  
Limiting distributions of discrete-time quantum walks on the square lattice
- 14:20 - 15:20 Shin-ichi Matsumura  
A vanishing theorem of Kollár-Ohsawa type on compact Kähler manifolds
- 15:30 - 16:10 Kenta Tottori  
Calabi's conjecture of the Kähler-Ricci soliton type
- 16:20 - 17:20 Masaharu Ishikawa  
Stable maps and hyperbolic volumes of 3-manifolds

## Abstract

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### Stabilities of rough curvature-dimension condition

Norihiko Suzuki

Lott-Villani and Sturm defined the curvature-dimension condition on metric measure spaces. Bonciocat-Sturm introduced the rough curvature dimension condition as a generalization of the condition by taking into account the roughness parameter. This condition characterizes the rough lower Ricci curvature bound and upper dimensional bound of metric measure spaces. They proved that this condition is stable under convergence with respect to the transportation distance in the case where the roughness parameter converges to 0. We prove the stability in the case where the roughness parameter converges to a nonzero real number. This talk is based on the joint work with Ryunosuke Ozawa in Kyoto University.

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### Strong algebraization of fixed point property

Masato Mimura

We obtain a purely algebraic criterion for fixed point properties on Banach spaces under relative fixed point properties. This is a strengthening of Shalom's algebraization in ICM 2006, in the sense of that we completely remove any form of "bounded generation" assumption. Several applications may, in addition, be discussed.

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### Limiting distributions of discrete-time quantum walks on the square lattice

Takashi Komatsu

We study discrete-time quantum walks on the square lattice. The notion of quantum walks was introduced by Y. Aharonov et al. as a quantum analogue of the classical random walks. Recently, quantum walks have been intensively studied in connection with quantum computing and quantum physics. In this talk, we will propose a model of discrete-time quantum walks on the square lattice without localization and give its limit distribution. Furthermore, we see that the Konno function appears as the density function with respect to radial direction in our quantum walk and give an another expression of our result from the view of the quantum information.

## **A vanishing theorem of Kollár-Ohsawa type on compact Kähler manifolds**

Shin-ichi Matsumura

In the last decades, it has been one of the important subjects in algebraic geometry to study the variation of the cohomology groups of vector bundles for deformations of algebraic varieties. In this talk, I would like to consider the higher direct images (which can be seen as such variations) for deformations of Kähler varieties parametrized by analytic spaces, and give a Hodge type decomposition for them by using the theory of harmonic integrals developed by Takegoshi. As an application, I will prove a vanishing theorem of Kollár-Ohsawa type in combined with the  $L^2$ -method for the  $\bar{\partial}$ -equation. It gives an affirmative answer for the conjecture posed by Fujino.

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## **Calabi's conjecture of the Kähler-Ricci soliton type**

Kenta Tottori

In this talk, we discuss Calabi's equation of the Kähler-Ricci soliton type on a compact Kähler manifold. This equation was introduced by Zhu as a generalization of Calabi's conjecture. We give necessary and sufficient conditions for the unique existence of a solution for this equation on a compact Kähler manifold with a holomorphic vector field which has a zero point. We also consider the case of a nowhere vanishing holomorphic vector field, and give sufficient conditions for the unique existence of a solution for this equation.

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## **Stable maps and hyperbolic volumes of 3-manifolds**

Masaharu Ishikawa

A stable map plays an important role when we study topology of manifolds. For example, a Morse function is a stable map from a manifold to the real line, which is used to obtain the information of homology groups of manifolds. We study 3-manifolds by using stable maps from them to the real plane. It is known by O.Saeki that there exists a stable map without certain singular fibers if and only if the Gromov norm of the 3-manifold is zero, i.e., it is a graph manifold. In the study of complexity of 3-manifolds, F.Costantino and D.Thurston used the idea of identifying the Stein factorization of a stable map with a shadow of the 3-manifold. Note that the above singular fibers correspond to the vertices of the shadow. We prove that there exists a stable map corresponding to a given shadow without changing the number of such singular fibers. As a consequence, we obtain an estimation of the Gromov norm of the 3-manifold by the minimal number of such singular fibers. This is a joint work with Yuya Koda in Hiroshima University.

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