Topics in Differential Geometry and its Discretizations

January. 10-12. 2015

Seminar room (2nd floor), WPI-AIMR, Main Building,
Tohoku University, Sendai, Japan.

Program & Abstract

Jan.10

12:00–12:50 Registration

13:00–14:00

James R. Lee (University of Washington)

Title: *Eigenvalues of the Laplacian, multi-flows, and metric uniformization.*

Abstract: The study of eigenvalues of the Laplace operator on graphs and manifolds has a long and rich history. In the Riemannian setting, conformal uniformization is a powerful tool for understanding the spectrum of the Laplace-Beltrami operator on surfaces. Even though graphs do not have an inherent conformal structure, we will introduce a form of metric uniformization that allows one to deform the underlying geometry by variational methods.

The uniformization approach allows us to resolve conjectures of Spielman and Teng about the spectrum of planar graphs and their generalizations. In particular, one can provide formal guarantees on the efficacy of widely-used spectral algorithms for graph partitioning. This answers positively a question of Gromov on whether there is a generalization of conformal techniques to minor-closed families of graphs that recovers the separator theorem of Alon, Seymour, and Thomas. One can show that the theorems for graphs actually imply corresponding results for surfaces. Thus as a special case, we recover Korevaar's bounds on the spectrum of compact surfaces.

14:10–15:10

James R. Lee (University of Washington)

Title: *Multi-way spectral partitioning and higher-order Cheeger inequalities.*

Abstract: A basic fact in spectral graph theory is that the number of connected components in an undirected graph is equal to the multiplicity of the eigenvalue zero in the Laplacian matrix of the graph. Cheeger 's inequality and its variants

provide a robust version of this fact for multiplicity 2. It has been conjectured that an analogous characterization holds for higher multiplicities: There are k small eigenvalues if and only if the vertex set of the graph can be partitioned into k sets with small boundary.

In the setting of mathematical physics, Simon and Hoegh-Krohn (1972) asked whether every hyper-bounded Markov operator has a spectral gap. Much later, Laurent Miclo showed that a positive answer to the first conjecture yields a positive answer to the second. In joint work with Oveis-Gharan and Trevisan, we prove Miclo's higher-order Cheeger conjecture. The proof is based on an eigenfunction localization method using random partitions of metric spaces. The argument is constructive and yields an algorithm for multi-way spectral partitioning that appears to work well in practice. Moreover, the study of higher-order Cheeger inequalities has some striking applications in computational complexity theory.

15:10–15:40 Tea break

15:40–16:40

Masato Mimura (Tohoku University)

Title: *Verifiable expander.*

Abstract: We say an infinite connected graph Gamma is a verifiable expander if any finite connected graph Lambda which "locally looks the same as Gamma" is an expander. That means, there exists a universal bound from below of the first positive Laplace eigenvalues of such Lambda. We will discuss a recent development on this notion.

16:50–17:50

Atsushi Kasue (Kanazawa University)

Title: *Kuramochi boundaries of transient networks.*

Abstract: We study functions of finite Dirichlet sum on a transient network and show that a family of the Dirichlet forms induced on the boundaries of finite subnetworks converges in a certain variational sense to the Dirichlet form induced by the random walk on the Kuramochi boundary of the network, if the subnetworks increase and exhaust the whole network.

18:00–21:00 Banquet (5th floor)

Jan.11

9:30–10:30

Hiroki Sako (Niigata University)

Title: *Sequence of finite metric spaces : amenability and connectivity*

Abstract: I will give a talk on discrete metric spaces, especially paying attention on disjoint unions of finite metric spaces. Among them, one having high connectivity is called an expander sequence. The item attracts not only mathematicians but also computer scientists. The objects relate to construction of networks and error correcting codes. We will focus on mathematical structure of uniformly locally finite metric spaces. It has been already known that a space cannot satisfy both of the following:

i) Coarse amenability,

ii) Containing an expander sequence.

I would like to show that every uniformly locally finite metric space has exactly one of i) or a little weaker condition than ii). In the proof, a Banach space of continuous linear operators plays a key role.

10:40–11:40

Hiroshi Hirai (University of Tokyo)

Title: *Some combinatorial optimization problems related to metric spaces of non-positive curvature*

Abstract: The max-flow min-cut theorem, due to Ford-Fulkerson, says that in a network with terminal pair s,t, the maximum value of an (s,t)-flow is equal to the minimum capacity of an (s,t)-cut. This theorem is one of most fundamental theorems in combinatorial optimization and graph theory. In this talk, we will discuss some generalizations of this theorem, and explain interesting connections to nonpositively-curved metric spaces.

(i) Multicommodity flow problem:

There are several generalizations of the max-flow min-cut theorem to “ multiflow ” settings. For a class of multiflow problems, the dual problem, which corresponds to “ min-cut ” in the single flow setting, is formulated as a location problem on a CAT(0) B_2 complex, called a "folder complex". The study of this formulation has brought significant contributions to several unsolved problems in multiflow theory.

(ii) Multifacility location problem (0-extension problem):

Problems of locating several facilities on graphs (or metric spaces) have numerous applications in operation research, machine learning and computer vision. Multifaclity location problem (or 0-extension problem) is an important special case where the cost of a location is given by a linear function on distances between facilities. The computation complexity of this problem depends on the underlying graph G. Recently the following dichotomy theorem was established:

If G is an "orientable modular graph", then the problem is polynomial time solvable (Hirai SODA13). Otherwise it is NP-hard (Karzanov 98).

The idea behind the proof of the polynomial time solvability is formulating the problem as an optimization problem on a certain metrized complex constructed from orientable modular graph G. It turned out that this complex is an "orthoscheme complex " introduced by Brady-McCammond in geometric group theory.

Joint with J. Chalopin, V. Chepoi, and D. Osajda, we showed that several non-positively curved complexes are constructed from orientable modular graphs. Examples include CAT(0)-cube complexes and Euclidean buildings of type C.

13:00–14:00, 14:10–15:10

Konrad Polthier (Freie Universität Berlin)

Title: *Discrete Minimal Surfaces based on Catmull-Clark Finite Elements I & II*

Abstract: We present a novel concept for discrete minimal surfaces based on Catmull-Clark finite elements. These surfaces are globally C1 though determined by a discrete control grid. We will report about the theoretical setup and provide several examples of these discrete minimal surfaces. A special advantage of this new discretization scheme is the uniform integration of discrete differential geometry, finite element analysis and computer aided design. Joint work with Anna Wawrzinek.

15:10–16:00 Tea break & Poster session

16:00–16:30

Masashi Yasumoto (Kobe University)

Title: *Discretization of linear Weingarten surfaces with Weierstrass-type representations*

Abstract: In this talk we discretize linear Weingarten surfaces in non-Euclidean spaceforms given by Weierstrass-type representations. In particular, discretizations of maximal surfaces in Lorentz-Minkowski 3-space and linear Weingarten surfaces of Bryant and Bianchi type are introduced. Furthermore, we compare smooth, discrete and semi-discrete surfaces with the same curvature conditions. This talk is partly

based on the joint work with Wayne Rossman, and will be highly related to the talk by Wayne Rossman that follows it.

16:40–17:40

Wayne Rossman (Kobe University)

Title: *Discretization of general linear Weingarten surfaces*

Abstract: Continuing from the previous talk by Masashi Yasumoto, we will consider how to discretize a more general class of surfaces, including those that do not have Weierstrass-type representations. We will see how general discrete linear Weingarten surfaces can be defined using constant conserved quantities of associated flat connections as a tool. Then, since smooth linear Weingarten surfaces typically have singularities (such as cuspidal edges, swallowtails, cuspidal cross-caps, etc), we will examine how to define corresponding singularities in the discretized case.

Jan. 12

9:30–10:30

Kenji Kajiwara (Kyushu University)

Title: *Integrable Discrete Deformations of Discrete Curves*

Abstract: We present some results on integrable discrete deformations of space/plane discrete curves in various settings, including: (i) isoperimetric deformation of plane curves (discrete mKdV equation) (ii) conformal deformation of plane curves (discrete Burgers equation) (iii) torsion-preserving deformation of space discrete curves of constant torsion (discrete mKdV and discrete sine-Gordon equation) (iv) curvature-preserving deformation of space discrete curves of constant curvature (discrete mKdV and discrete sine-Gordon equation).

10:40–11:40

Ken-ichi Maruno (Waseda University)

Title: *Discretization of integrable systems and self-adaptive moving mesh schemes*

Abstract: In recent years, we have investigated integrable discretizations of integrable nonlinear partial differential equations (PDEs) whose solutions possess singularities. For PDEs in this class, we obtained various discrete integrable systems with self-adaptive moving mesh which can be powerful tools for numerical simulations of nonlinear PDEs. One of keys to construct self-adaptive moving mesh schemes is a discretization of hodograph transformations which are transformations between Lagrangian description and Eulerian description. In a geometric point of view, self-adaptive moving mesh schemes can be interpreted as Eulerian description of the motion of discrete curves. In this talk, I will review recent results of discrete integrable systems with self-adaptive moving mesh.

11:50–12:50

Shoichi Fujimori (Okayama University)

Title: *Degenerate limits of triply periodic minimal surfaces of genus 3*

Abstract: We consider limits of triply periodic minimal surfaces in Euclidean 3-space. We prove that some important examples of singly or doubly periodic minimal surfaces can be obtained as limits of triply periodic minimal surfaces. This is joint work with Norio Ejiri and Toshihiro Shoda.

Poster Session

Title: *Discrete-time quantum walk on the square lattice*

Author: Takashi Komatsu* (Tohoku University)

Abstract: We study discrete-time quantum walks on graphs. The notion of discrete- time quantum walks was introduced as a quantum version of random walks. Recently, quantum walks have been intensively studied in connection with quantum computing and quantum physics.

In this poster, we will propose a model of discrete-time quantum walks on the square lattice without localization and give the limiting distribution of our quantum walk. Next, we see that the Konno function appears as the density function with respect to radial direction in our quantum walk.

Title: *Translating soliton in arbitrary codimension*

Author: Keita Kunikawa* (Tohoku University)

Abstract: We study the translating solitons of mean curvature flow. Although many authors study translating solitons in codimension one, there are few references and examples for higher codimensional cases except for Lagrangian translating solitons. Hence we observe non-trivial examples of translating solitons in arbitrary codimension. We will see that they have the property called parallel principal normal (PPN). Conversely, we classify the complete translating solitons with PPN.

Title: *Hodge-Kodaira Decomposition of Evolving Neural Networks*

Authors: Keiji Miura* (Tohoku University), Takaaki Aoki (Kagawa University)

Abstract: Here we applied the Hodge-Kodaira decomposition, a topological method, to an evolving neural network model in order to characterize its loop structure. By controlling a learning rule parametrically, we found that a model with an STDP-rule, which tends to form paths coincident with causal firing orders, had the most loops. Furthermore, by counting the number of global loops in the network, we detected the inhomogeneity inside the chaotic region, which is usually considered intractable.

Title: *Neural Implementation of Shape-Invariant Touch Counter Based on Euler Calculus*

Authors: Keiji Miura* (Tohoku University), Kazuki Nakada (University of Electro-Communications)

Abstract: Here we propose a fully parallelized algorithm for a shape-invariant touch counter for 2-D pixels. The number of touches is counted by the Euler integral, a generalized integral, in which a connected component counter (Betti number) for the binary image was used as elemental module. The proposed circuit architecture embodies the Euler integral in the form of recurrent neural networks for iterative vector operations. Our parallelization can lead the way to Field-Programmable Gate Array or Digital Signal Processor implementations of topological algorithms with scalability to high resolutions of pixels.

Title: *The DPW method for discrete constant mean curvature surfaces in Riemannian spaceforms*

Author: Yuta Ogata* (Kobe University)

Abstract: Bobenko and Pinkall discretized constant mean curvature (CMC) surfaces in Euclidean 3-space in terms of Lax representations. Applying matrix factorizing theorems, Hoffmann gave a generalized Weierstrass-type representation for discrete CMC surfaces in the sense of Bobenko and Pinkall. In this poster, we will explain that such construction can be extended to discrete CMC surfaces in other Riemannian spaceforms. This poster is based on the joint work with M. Yasumoto.

Title: *Phase transition property of l_p-product spaces*

Author: Ryunosuke Ozawa* (Tohoku University)

Abstract: We consider metric measure spaces X_n close to a one-point metric measure space if for any 1-Lipschitz function on X is close to a constant function. This phenomenon is called the measure concentration. The ∞-dissipation property is opposite from the measure concentration and means that the metric measure spaces

disperse into many small pieces far apart each other. A sequence $\{X_n\}_{n=1}^{\infty}$ of metric measure spaces has the phase transition property if there exists a sequence $\{c_n\}_{n=1}^{\infty}$ satisfying the following (1) and (2). (1) For any sequence $\{t_n\}_{n=1}^{\infty}$ with $t_n/c_n \to 0$ as $n \to \infty$, the scaled metric measure space $t_n X_n$ close to the one-point metric measure space as $n \to \infty$. (2) For any sequence $\{t_n\}_{n=1}^{\infty}$ with $t_n/c_n \to +\infty$ as $n \to \infty$, the sequence $\{t_n X_n\}_{n=1}^{\infty}$ ∞-dissipates. We call such a sequence $\{c_n\}_{n=1}^{\infty}$ a sequence of critical scale order. In this poster, we give a sequence of critical scale order of a sequence of the l_p-product spaces. This poster is based on a joint work with Takashi Shioya (Tohoku University).

Title: *Wave splitting solution for the FitzHugh-Nagumo equations*

Author: Tomoyuki Terada* (Tohoku University)

Abstract: Using numerical simulations, we discover a wave splitting solution for the FitzHugh-Nagumo equations under temporal switching between two states of mono- and bi-stable nonlinearities. Mono(bi)-stable nonlinearity has several unstable steady states and only one(two) stable steady state, respectively. To analyze the pattern formation dynamics containing the wave splitting solution, we define of a wave splitting solution describing the discovered solution.

Title: *Parallel surfaces of cuspidal edges*

Author: Keisuke Teramoto* (Kobe University)

Abstract: We investigate parallel surfaces of cuspidal edges. We give a criterion for the parallel surfaces of cuspidal edges to have swallowtail singularities. Moreover, we also clarify relations between singularities of parallel surfaces and differential geometric properties of initial cuspidal edges.

Title: *Final state problem for a system of nonlinear Schrödinger equations with three wave interaction*

Author: Kota Uriya* (Tohoku University)

Abstract: In this poster, we consider a system of nonlinear Schrödinger equations with three wave interaction. We derive the asymptotic behavior of a solution to the system by using a particular solution to a system of ordinary differential equations.