Translating solitons in arbitrary codimension

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Mean curvature flow

A family of isometric immersions of a Riemannian manifold

$$F : M^n \times [0, t_0) \to \mathbf{R}^{n+m}.$$

Deform the immersion by the mean curvature vector

$$\frac{\partial}{\partial t} F(p,t) = \vec{H}(p,t).$$

- MCF is the L^2-gradient flow of the area functional.
- In general, MCF has singularities.

In arbitrary codimension

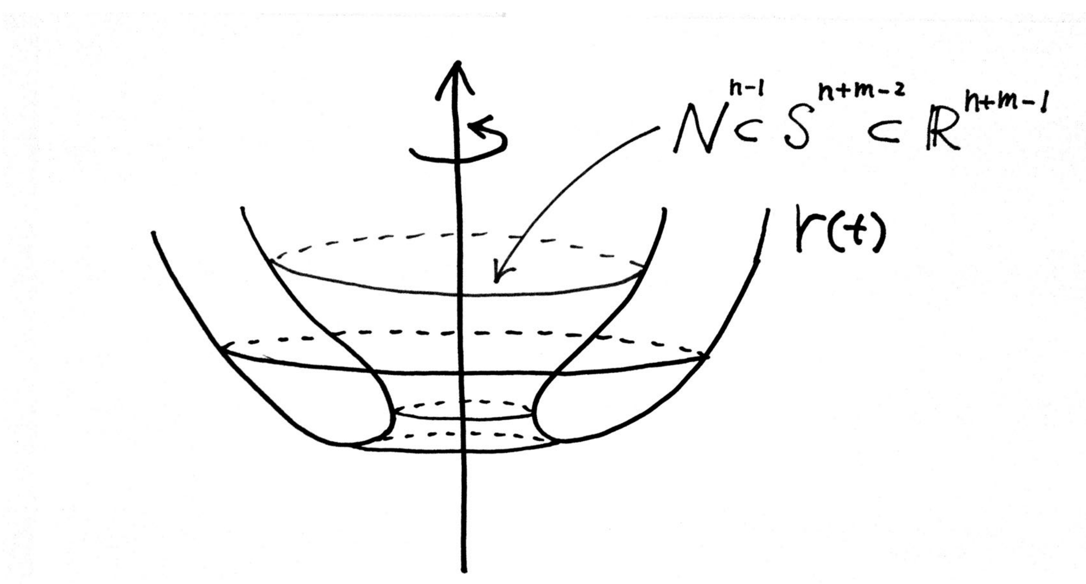
- Difficult to deal with.
- Singularities in arbitrary codimenson are less-known.

Translating soliton

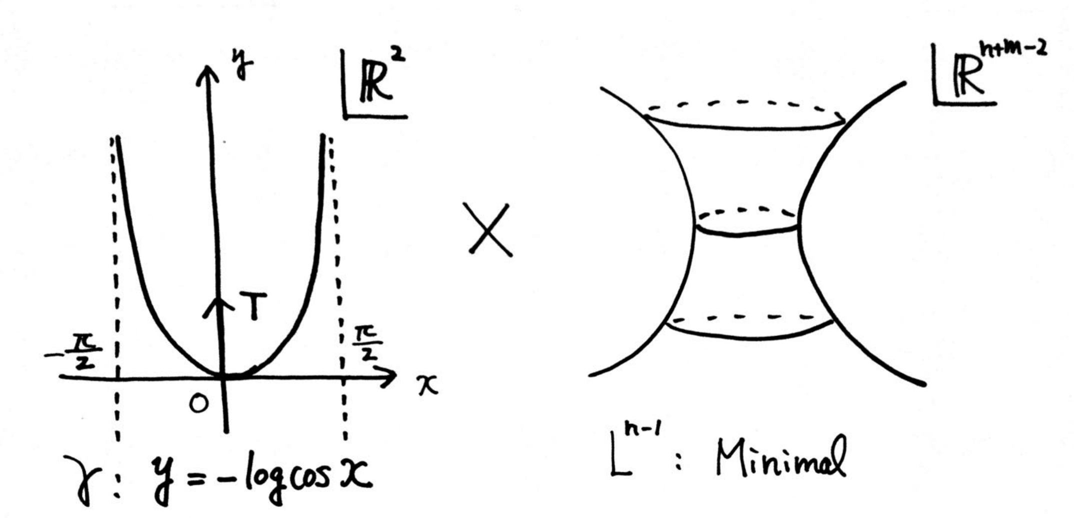
Translating soliton is defined by

$$\vec{H} = T^{\perp}, \quad T \in \mathbf{R}^{n+m}.$$

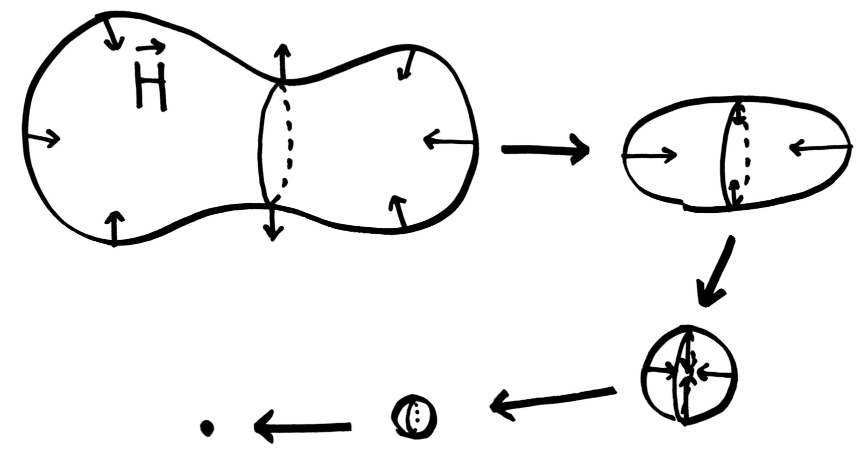
- A famous model of a singularity.
- Moving by translation in the direction of T under the MCF.



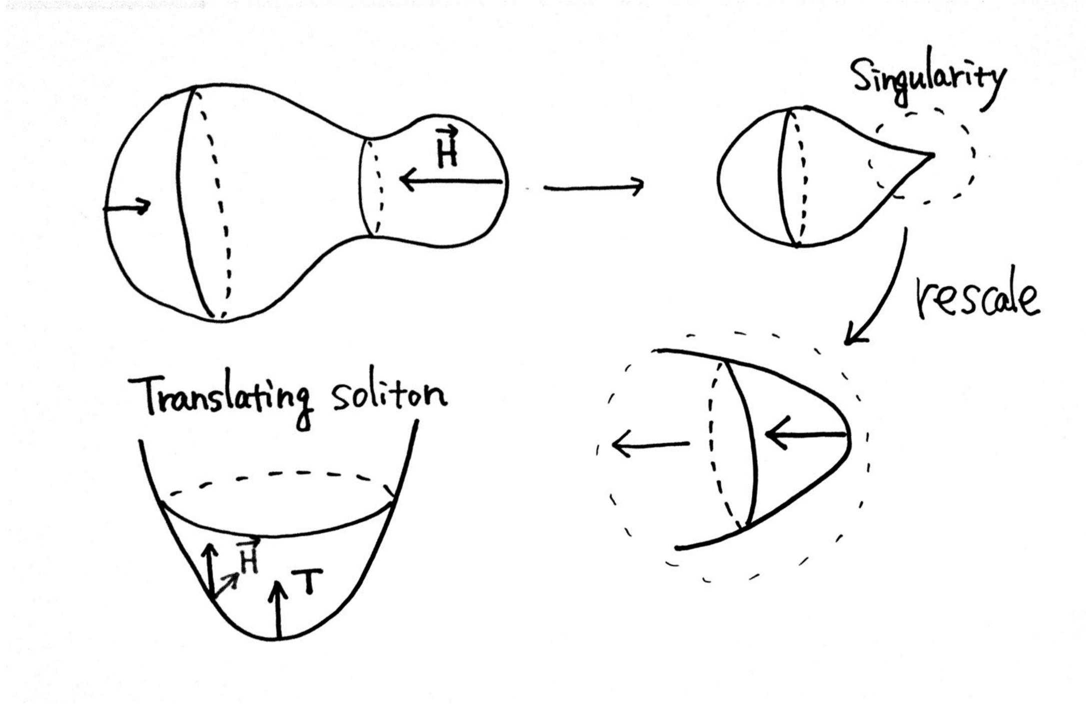
New example



Splitting of a translating soliton



Mean curvature flow



Singularity

New examples in arbitrary codimension

Let $N^{n-1} \subset \mathbf{S}(1)^{n+m-2} \subset \mathbf{R}^{n+m-1}$ be a complete minimal submanifold. Set $M^n := N^{n-1} \times \mathbf{R}$.

Theorem 1 (K., 2015) *Define the immersion*

$$F(q,t) := (tq, r(t)) \subset \mathbf{R}^{n+m},$$

where $r(t)$ is a solution of

$$\ddot{r}(t) = \left(1 + \dot{r}(t)^2\right)\left(1 - \frac{(m-1)\dot{r}(t)}{t}\right).$$

Then $F(M^n)$ is a translating soliton with $T = (0, \cdots, 0, 1)$.

Parallel principal normal

Principal normal is $\nu := \vec{H}/|\vec{H}|$. PPN is defined by $\nabla^{\perp}\nu \equiv 0$. Let $P := \langle B, \vec{H}\rangle$, where B is the second fundamental form.

Theorem 2 (K., 2015) $M^n \subset \mathbf{R}^{n+m}$: *complete translating soliton with PPN. If $|P^2|/|\vec{H}|^4$ attains its local maximum, then*

$$M^n = \gamma \times L^{n-1} \subset \mathbf{R}^2 \times \mathbf{R}^{n+m-2},$$

where γ is the grim reaper ($y = -\log\cos x$) and L^{n-1} is a complete minimal submanifold.