

# International Workshop on Special Geometry and Minimal Submanifolds

## Title and Abstract

- **Jurgen Berndt (King's College London)**

Title : Polar actions on symmetric spaces

Abstract : An isometric action of a connected Lie group on a Riemannian manifold is called polar if there exists a connected closed submanifold which meets each orbit of the action and intersects it orthogonally. Dadok established in 1985 a remarkable, and mysterious, relation between polar actions on Euclidean spaces and Riemannian symmetric spaces. Soon afterwards an attempt was made to classify polar actions on symmetric spaces. For irreducible symmetric spaces of compact type the final step of the classification has just been completed by Kollross and Lytchak. In the talk I want to focus on symmetric spaces of noncompact type. For actions of reductive groups one can use the concept of duality between symmetric spaces of compact type and of noncompact type. However, new examples and phenomena arise from the geometry induced by actions of parabolic subgroups, for which there is no analogy in the compact case. I plan to discuss the main difficulties one encounters here and some partial solutions.

- **Jaigyoung Choe (KIAS)**

Title : Compact minimal surfaces in  $S^3$

Abstract : The minimal surfaces in  $R^3$  can be constructed easily by using the Weierstrass representation formula. But there is no such formula in  $S^3$ , and so only a few families of minimal surfaces have been known to exist in  $S^3$ . In this talk we first introduce the minimal surfaces in  $S^3$  that have been constructed by Lawson(1970), by Karcher-Pinkall-Sterling(1988), and by Kapouleas-Yang(2010). Then we introduce newly constructed minimal surfaces(2013).

- **Jason Lotay (Univ. College London)**

Title : Stability, conifolds and  $G_2$  geometry

Abstract : Conifolds are manifolds which have asymptotically conical ends or have conical singularities. In the context of  $G_2$  geometry, key geometric questions for conifolds are intimately related to the spectrum of an elliptic operator on the cross-section of the cone. From this spectrum, based on the pioneering work of Joyce, one is led to define an integer invariant associated with the cone called the stability index. I will describe the connections that the stability index has to various problems for conifolds, with an especial focus on coassociative submanifolds in 7-manifolds with  $G_2$  holonomy, including: deformation theory, gluing problems and existence and uniqueness questions.

- **Young Jin Suh (Kyungpook Univ.)**

Title : Hypersurfaces with Isometric Reeb Flow in Hermitian Symmetric Spaces of rank 2

Abstract : In this talk, first we introduce the classification of homogeneous hypersurfaces in some Hermitian symmetric spaces of rank 1 or rank 2. In particular, we give a full expression of the geometric structures for hypersurfaces in complex two-plane Grassmannians  $G_2(C^{m+2})$  or in complex hyperbolic two-plane Grassmannians  $G_2^*(C^{m+2})$ .

Next by using the isometric Reeb flow we give a complete classification for hypersurfaces  $M$  in complex two-plane Grassmannians  $G_2(C^{m+2})$ , complex hyperbolic two-plane Grassmannians  $G_2^*(C^{m+2})$  and a complex quadric  $\mathbb{Q}^m$ .

- **Norio Ejiri (Meijo Univ.)**

Title : Ruled complex Lagrangian submanifolds of dimension two in  $\mathbb{C}^4$

Abstract : We construct examples of ruled complex Lagrangian submanifolds in  $\mathbb{C}^4$  as deformations of the complex Lagrangian cone in  $\mathbb{C}^4$

- **Hideya Hashimoto (Meijo Univ.)**

Title : On group of automorphisms of 6-submanifolds in the octonionons and its applications.

Abstract : We recall the fundamental facts about 6-submanifolds in the octonionons, and give the groups of automorphisms of its induced orthogonal almost complex structures on several 6-submanifolds. Also we give applications related to geometrical structures on the 6-submanifolds.

- **Kota Hattori (Univ. Tokyo)**

Title : A generalization of Taub-NUT deformations

Abstract : Taub-NUT metrics on  $C^2$  are complete Ricci-flat Kaehler metrics which are not flat. They are obtained by the Taub-NUT deformations of the Euclidean metric on  $C^2$  using an  $S^1$  action. Taub-NUT deformations are known to be defined for toric hyperKaehler manifolds, and deform ALE metrics to non-ALE metrics. In this talk, I explain a generalization of Taub-NUT deformations by using noncommutative Lie groups.

- **Tomoyuki Hisamoto (Nagoya Univ.)**

Title : Remarks on the extension problem for singular Hermitian metrics with semipositive curvature

Abstract : We give a new variant of  $L^2$ -extension theorem for the jets of holomorphic sections along submanifolds and discuss the relation between the extension problem of singular Hermitian metrics with semipositive curvature.

- **Kotaro Kawai (Tohoku Univ.)**

Title : Deformations of associative submanifolds in nearly parallel  $G_2$ -manifolds

Abstract : A  $G_2$ -manifold  $Y$  is called nearly parallel if its cone is a torsion-free  $\text{Spin}(7)$ -manifold. An associative submanifold  $M$  in  $Y$  is defined to be a 3-dimensional minimal submanifold which is related to the  $G_2$ -structure of  $Y$ .

It is known that Sasaki-Einstein 7-manifolds are nearly parallel  $G_2$ -manifolds, and special Legendrian 3-submanifolds are associative.

We study the infinitesimal deformations of an associative submanifold  $M$  in a nearly parallel  $G_2$ -manifold  $Y$ . We show the difference between associative and special Legendrian deformations in the Sasaki-Einstein case.

Then we study the homogeneous associative submanifolds in the 7-sphere  $S^7$ , and obtain some rigidity results.

- **Miyuki Koiso (Kyushu Univ. IMI)**

Title : Free boundary problem for surfaces with constant mean curvature

Abstract : We study embedded surfaces of constant mean curvature with free boundary in given supporting planes in the euclidean three-space. We assume that each considered surface meets the supporting planes with constant contact angle. These surfaces are characterized as equilibrium surfaces of the variational problem of which the total energy is the surface area and a wetting energy (that is a weighted area of the domains in the supporting planes bounded by the boundary of the considered surface) with volume constraint. An equilibrium surface is said to be stable if the second variation of the energy is nonnegative for all volume- preserving variations satisfying the boundary condition. We are interested in determining all (stable) solutions. At present in literature, only for some special cases, for example, the supporting planes are either just a single plane or two parallel planes and the wetting energy is nonnegative, all stable solutions are known. We discuss recent progress of this subject and show the space of solutions is not continuous with respect to the boundary condition.

- **Yoshihiko Matsumoto (Univ. Tokyo)**

Title : On the total CR Q-curvature and its variational properties

Abstract : Branson's Q-curvature in even-dimensional conformal geometry is a generalization of the Gauss curvature on Riemann surfaces, whose conformal transformation law is governed by a conformally invariant differential operator with leading term a power of the Laplacian. I will introduce its analog in CR geometry using the asymptotic expansion of a certain complete Kähler-Einstein metric with negative curvature, or more generally an asymptotically complex hyperbolic Einstein metric, with prescribed CR structure at the boundary at infinity. Moreover, both in conformal and CR geometries, the Q-curvature integrates to a global invariant that we call the total Q-curvature. I will discuss the recent progress on its first and second variations with emphasis in the CR case.

- **Hiroshi Tamaru (Hiroshima Univ.)**

Title : Left-invariant metrics on Lie groups and submanifold geometry

Abstract : Left-invariant Riemannian metrics on Lie groups have provided many interesting examples of homogeneous Einstein and Ricci soliton manifolds. In general, it is not easy to examine whether a given Lie group admits such distinguished metrics or not. In this talk, I will explain our approach from submanifold geometry. In particular, for three-dimensional

solvable Lie groups, the existence and nonexistence of left-invariant Ricci solitons have a nice correspondence with the geometry of cohomogeneity one actions on some noncompact symmetric space. I will also mention some higher-dimensional examples and a pseudo-Riemannian version.

- **Masaaki Umehara (TIT)**

Title : Zero mean curvature surfaces in Lorentz-Minkowski 3-space which change type across a light-like line

Abstract : It is well-known that space-like maximal surfaces and time-like minimal surfaces in Lorentz-Minkowski 3-space  $R_1^3$  have singularities in general. They are both characterized as zero mean curvature surfaces. We are interested in the case where the singular set consists of a light-like line, since this case has not been analyzed before. As a continuation of a previous work by the authors, we give the first example of a family of such surfaces which change type across the light-like line. As a corollary, we also obtain a family of zero mean curvature hypersurfaces in  $R_1^{n+1}$  that change type across an  $(n-1)$ -dimensional light-like plane. This is a joint work with S. Fujimori, Y.-W. Kim, S.-E. Koh, W. Rossman, H. Shin, K. Yamada and S.-D. Yang.

### Short talk

- **Eunmi Pak (Kyungpook Univ.)**

Title : Parallelism with respect to the generalized Tanaka-Webster connection of Hopf hypersurfaces in complex two-plane Grassmannians

Abstract : In this paper, we introduce new notions in relation to the generalized Tanaka-Webster connection for a hypersurface in complex two-plane Grassmannians and give classifications of Hopf hypersurfaces in complex two-plane Grassmannians.

- **Changhwa Woo (Kyungpook Univ.)**

Title : Generalized Tanaka-Webster Reeb parallel Ricci tensors of Hopf hypersurfaces in complex two-plane Grassmannians

Abstract : There are several kinds of classification problems for real hypersurfaces in complex two-plane Grassmannians  $G_2(C^{m+2})$ . Among them, Suh classified a Hopf hypersurface  $M$  in  $G_2(C^{m+2})$  with Reeb parallel Ricci tensor in Levi-Civita connection. In this paper, we introduce a new notion of generalized Tanaka-Webster Reeb parallel Ricci tensor for  $M$  in  $G_2(C^{m+2})$ . Related to such a notion, we give some characterizations for a real hypersurface of Type (A) in  $G_2(C^{m+2})$ .

- **Toru Kajigaya (Tohoku Univ., King's College London)**

Title : On the minimality of normal bundles and austere submanifolds

Abstract : The notion of austere submanifolds in a Riemannian manifold was first introduced by Harvey and Lawson to construct special Lagrangian submanifolds in the complex Euclidean space  $C^n$ . They proved that a submanifold in the Euclidean space  $R^n$  is austere if and only if the normal bundle is special Lagrangian in the tangent bundle over  $R^n$  which is naturally regarded as  $C^n$ . However, the geometrical interpretation of austere submanifolds in general Riemannian manifolds is unknown except a few cases. In this talk, we generalize their result as a context of minimal normal bundles in tangent bundles equipped with Sasaki metric, and give a characterization of austere submanifolds in the real space forms. Moreover, we investigate extrinsic properties of Lagrangian normal bundles. In particular, we discuss the minimality of normal bundles in the tangent bundles over the complex space forms, and the Hamiltonian minimality of normal bundles over isoparametric submanifolds in  $R^n$ .

- **Satoshi Ueki (Tohoku Univ.)**

Title : Stability of non-compact minimal Lagrangian submanifolds in Kähler manifolds and  $L^2$  harmonic 1-forms

Abstract : It is known that a compact Lagrangian stable minimal Lagrangian submanifold in a Kähler submanifold with positive Chern class has no nontrivial harmonic 1-forms. On the other hand, it is also known that a complete non-compact stable minimal hypersurface in a Riemannian manifold with non-negative sectional curvature has no non-trivial  $L^2$  harmonic 1-forms. In this talk, we will consider the relation between stability of complete non-compact minimal Lagrangian submanifolds in a Kähler submanifold and nonexistence of  $L^2$  harmonic 1-forms.