1st Italian-Japanese workshop on geometric properties for parabolic and elliptic PDE's

Abstract

June 15-19, 2009

Tohoku University, Sendai, Japan

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ABSTRACTS

Drift-diffusion system in critical space

Takayoshi Ogawa
Tohoku University, Japan

Abstract. We consider the following Cauchy problem for drift-diffusion system:

$$\begin{cases} \partial_t \rho - \Delta\rho + \nabla \cdot (\rho \nabla \psi) = 0, & x \in \mathbb{R}^2,\ t > 0, \\ -\Delta\psi = \kappa\rho, & x \in \mathbb{R}^2,\ t > 0, \\ \quad \rho(0, x) = \rho_0(x) > 0, & x \in \mathbb{R}^2. \end{cases} \tag{1}$$

where ρ denotes the density function and ψ is unknown potential determined by the density. The constant κ gives two different situations. Namely when $\kappa < 0$ the system has a large data global solution while the first case, the solution may blows up in a finite time. Blanchet-Dobeault-Perthame [2] give the existence of global solution and discussed the asymptotic behavior using the entropy functional.

We give the following existence result. [1]

Theorem 1 *Suppose that* $\rho_0 \geq 0$, $\rho_0 \in L^1(\mathbb{R}^2)$ *and* $\log(1 + |x|)\rho_0(x) \in L^1(\mathbb{R}^2)$ *with the restriction*

$$\|\rho_0\|_1 < 8\pi.$$

Then there exists a global solution (ρ, ψ) *of* (1) *with*
$\rho \in C([0, \infty); L^1(\mathbb{R}^2)) \cap C^1((0, \infty); L^1(\mathbb{R}^2)) \cap C((0, \infty); L^{4/3}(\mathbb{R}^2))$
(1) the solution is unique, (2) the solution depends on the initial data continuously and (3) the total mass $\|\rho(t)\|_1$ *is conserved.*

The existence of the solution we derive the a priori estimate for the solution by the Entropy functional:

$$W(t) = \int_{\mathbb{R}^2} (1 + \rho(t)) \log(1 + \rho(t))dx - \frac{1}{2} \int_{\mathbb{R}^2} \rho\psi(t)dx.$$

We also use the Brezis-Merle type inequality.

Proposition 2 [3] *Let* $\Omega \subset R^2$ *be a bounded domain with smooth boundary. Let* w *be a solution of* $-\Delta w = g$ *in* Ω *If* $\|g\|_{L^1(\Omega)} < 4\pi$ *then*

$$\int_\Omega \exp(|w|)\, dx \leq \frac{C|\Omega|}{4\pi - \|g\|_{L^1(\Omega)}} \exp(\sup_{\partial\Omega} |w|)$$

The positive solution with $\|\rho_0\|_1 > 8\pi$ with a certain weight condition is known to be blow-up in a finite time and for $\|\rho_0\|_1 = 8\pi$ the behavior of solution depends on each initial data.

[1] This is a joint work with Toshitaka Nagai (Hiroshima Univ)

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Bounds for eigenfunctions of the Laplacian on non-compact Riemannian manifolds

Andrea Cianchi
Università di Firenze, Italy

Abstract. Estimates for the supremum and for Lebesgue norms of eigenfunctions of the Laplace operator on (possibly) non-comapct Riemannian manifolds are derived in terms of isocapacitary inequalities. These inequalities relate the capacity of subsets of the relevant manifolds to their measure, and improve the more classical isoperimetric inequalities. This is a joint work with V.G.Maz'ya.

An eigenvalue problem related to blowing-up solutions for the Brezis-Nirenberg equation

Futoshi Takahashi
Osaka City University, Japan

Abstract. Let $\Omega \subset \mathbb{R}^N (N \geq 4)$ be a smooth bounded domain, $p = (N+2)/(N-2)$, $c_0 = N(N-2)$ and $\varepsilon > 0$ is a parameter.

In this talk, we are concerned with some spectral properties of least energy solutions u_ε to the problem

$$(P_{\varepsilon,k}) \begin{cases} -\Delta u = c_0 u^p + \varepsilon k(x) u & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

when $\varepsilon > 0$ is small, where $k \in C^2(\overline{\Omega}),\ k > 0$ is a given function.

Let us consider the linearized eigenvalue problem around the least energy solution u_ε to $(P_{\varepsilon,k})$:

$$(E_{\varepsilon,k}) \begin{cases} -\Delta v_{i,\varepsilon} = \lambda_{i,\varepsilon} \left(c_0 p u_\varepsilon^{p-1} + \varepsilon k(x) \right) v_{i,\varepsilon} & \text{in } \Omega, \\ v_{i,\varepsilon} = 0 & \text{on } \partial\Omega, \\ \|v_{i,\varepsilon}\|_{L^\infty(\Omega)} = 1 & \end{cases}$$

for $i \in \mathbb{N}$. Under some assumptions of the coefficient function k, we prove precise asymptotic estimates for the first $(N+2)$ eigenvalues and eigenfunctions of $(E_{\varepsilon,k})$ as $\varepsilon \to 0$.

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On a new kind of convexity for solutions of parabolic problems

Paolo Salani
Università di Firenze, Italy

Abstract. I will present the results of a joint paper with Kazuhiro Ishige.

We consider the following problem

$$\begin{cases} \partial_t u = F(x, u, Du, D^2 u) & \text{in } \mathcal{R} \times (0, +\infty) \\ u(x, 0) = u_0(x) & x \in \mathcal{R} \\ u(x, t) = 0 & (x, t) \in \partial\Omega \times (0, \infty) \\ u(x, t) = g(t) & (x, t) \in K \times (0, \infty) \end{cases} \tag{1}$$

where $\partial_t = \partial/\partial t$, Ω is an open convex set and K is a non-empty compact convex set such that $K \subset \Omega \subseteq \mathbb{R}^N$, $N \geq 1$, $\mathcal{R} = \Omega \setminus K$, F is a proper elliptic operator and u_0 and g are sufficiently regular functions, such that $u_0(x) = g(0)$ for $x \in \partial K$ and $u_0(x) = 0$ for $x \in \partial\Omega$.

Then we investigate on concavity properties of solutions to (1), when the initial datum u_0 is a quasi-concave function, i.e. a function whose superlevel sets are all convex. We prove that, under suitable conditions on the operator F, the solution u is spatially quasi-concave at any fixed time $t \geq 0$. Moreover we introduce the notion of *parabolic quasi-concavity*, a quasi-concavity property involving time and space jointly, and we give conditions on F that force u to be parabolically quasi-concave, when the initial datum u_0 identically vanishes.

Singular positive solutions for a fourth order elliptic problem in $\mathbf{R}^N$

Tatsuya Watanabe
Osaka City University, Japan

Abstract. In this talk, we study the following fourth order elliptic problem:

$$\begin{cases} \Delta^2 u - c_1 \Delta u + c_2 u = u^p + \kappa \sum_{i=1}^m \alpha_i \delta_{a_i} \text{ in } \mathcal{D}'(\mathbb{R}^N), \\ u(x) > 0, \ u(x) \to 0 \text{ as } |x| \to \infty, \end{cases} \tag{1}$$

where $N \geq 5$, $\kappa > 0$, $m \in \mathbb{N}$, $\alpha_i > 0$ and c_1, $c_2 > 0$. Here we denote by δ_{a_i} the delta function supported at $a_i \in \mathbb{R}$. We assume further that

$$1 < p < \frac{N}{N-4} \text{ and } c_1^2 - 4c_2 \geq 0.$$

For simplicity, we write $L = \Delta^2 - c_1 \Delta + c_2 I$ and $f(x) = \sum_{i=1}^m \delta_{a_i}$. The operator L is related to the Paneitz-Branson operator, which is conformally invariant on smooth compact Riemannian manifolds and can be seen as a natural extension of the Laplace-Beltrami operator.

We notice that the condition $c_1^2 - 4c_2 \geq 0$ enables us to rewrite (1) by the following elliptic system:

$$-\Delta u + t_1 u = v, -\Delta v + t_2 v = u^p + \kappa f.$$

Here $t_1 = \dfrac{c_1 - \sqrt{c_1^2 - 4c_2}}{2}$ and $t_2 = \dfrac{c_1 + \sqrt{c_1^2 - 4c_2}}{2}$. We note that $0 < t_1 \leq t_2$. In this situation, we obtain the following result.

Theorem 1 *There exists $\kappa^* \in (0, \infty)$ satisfying the following properties:*
(i) Problem (1) has no solution for all $\kappa > \kappa^*$.
(ii) For any $\kappa \in (0, \kappa^*)$, there exist two solutions u_κ and u^κ of (1) such that
(a) u_κ, $u^\kappa \in C^4(\mathbb{R}^N \setminus \bigcup_{i=1}^m \{a_i\})$ and $u_\kappa(x) < u^\kappa(x)$ for $x \in \mathbb{R}^N \setminus \bigcup_{i=1}^m \{a_i\}$.
(b) $u_\kappa(x)$, $u^\kappa(x) = O(\frac{1}{|x-a_i|^{N-4}})$ as $|x - a_i| \to 0, (i = 1, 2, \cdots, m)$.
(c)

$$u_\kappa(x), u^\kappa(x) = \begin{cases} O(|x|^{-\frac{N-3}{2}} e^{-\sqrt{t_1}|x|}) \text{ as } |x| \to \infty & \text{if } t_1 = t_2 \\ O(|x|^{-\frac{N-1}{2}} e^{-\sqrt{t_1}|x|}) \text{ as } |x| \to \infty & \text{if } t_1 < t_2. \end{cases}$$

This is a joint work with Tokushi Sato (Tohoku University).

Symmetry and stability in an overdetermined problem for the Green's function

Virginia Agostiniani
SISSA Trieste, Italy

Abstract. We consider in the plane the problem of reconstructing a domain from the normal derivative of its Green's function. By means of the theory of analytic functions, we first obtain (also non-spherical) symmetry results and secondly derive stability estimates of polynomial type in Hölder norms.

Hardy-Sobolev critical elliptic problems with multiple singular points

Takahiko Chujo
Hiroshima University, Japan

Abstract. In this talk we consider the following nonlinear elliptic equation with multiple singular points $P_1 \cdots, P_m$

$$-\Delta u = \sum_{i=1}^{m} \frac{u^{2^*(s_i)-1}}{|x-P_i|^{s_i}}, \tag{$*$}$$

where

$$n \geqq 4, \quad 0 < s_i < 2 \quad \text{and} \quad 2^*(s_i) := \frac{2(n-s_i)}{n-2} \ (i = 1, \cdots, m).$$

The exponent $2^*(s)$ is said to be *critical* in view of the Hardy-Sobolev inequality . A natural strategy is using the Mountain-Pass lemma with the help of a suitable functional to find positive solutions of $(*)$. However, the Palais-Smale condition does not hold in this critical case. Therefore, the main ingredient in this analysis comes from the invariance of

$$\|\nabla u\|_{L^2(\mathbb{R}^n)} \qquad \text{and} \qquad \int_{\mathbb{R}^n} \frac{|u|^{2^*(s)}}{|x|^s} dx,$$

under scaling $u(x) \ \mapsto \ r^{(n-2)/2} u(rx)$.

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Nonlinear scalar field equations in $\mathbf{R}^N$ -a mountain pass approach-

Norihisa Ikoma
Department of Pure and Applied Mathematics,
Graduate School of Fundamental Science and Technology,
Waseda University, Tokyo, Japan

Abstract. In this talk, we consider the existence of radially symmetric solutions of the following nonlinear scalar field equations:

$$\begin{cases} -\Delta u = g(u) & \text{in } \mathbf{R}^N, \\ u \in H^1(\mathbf{R}^N), \end{cases} \tag{1}$$

where $N \geq 2$ and $g(s) : \mathbf{R} \to \mathbf{R}$ is continuous and odd.

Berestycki and Lions (1983) and Berestycki, Gallouët and Kavian (1983) studied this problem and they obtained very general existence results. To prove their existence result, they considered the minimization or minimax methods for the constrained problem.

We remark that the solutions of (1) are characterized as critical points of the following functional $I(u) \in C^1(H^1_r(\mathbf{R}^N), \mathbf{R})$:

$$I(u) = \frac{1}{2}\int_{\mathbf{R}^N} |\nabla u|^2 dx - \int_{\mathbf{R}^N} G(u) dx,$$

where $H^1_r(\mathbf{R}^N)$ is the space of radially symmetric functions in $H^1(\mathbf{R}^N)$. It is a natural question whether it is possible to obtain solutions of (1) via unconstrained functional $I(u)$ (e.g. the mountain pass theorem).

The aim of this talk is to give an affirmative answer to this question and to give a generalization of the results of Berestycki et al. for $N = 2$. More precisely, we introduce a new method to generate a Palais–Smale sequence with an additional property related to the Pohozaev identity:

$$\frac{N-2}{2}\int_{\mathbf{R}^N} |\nabla u|^2 dx - N\int_{\mathbf{R}^N} G(u) dx = 0.$$

This is a joint work with J. Hirata and K. Tanaka (Waseda University).

Stable patterns for shadow systems and a nonlinear “hot spots” conjecture

Yasuhito Miyamoto
Department of Mathematics, Tokyo Institute of Technology, Meguro-ku, Tokyo, 152-8551, Japan

Abstract. In 1975 Chafee has shown that if a steady state of a scalar reaction-diffusion equation with the Neumann boundary condition is stable, then it should be a homogeneous function. In 1978 Casten-Holland and in 1979 Matano have independently shown that the same conclusion holds for a convex domain in $\mathbb{R}^N$. In 1994 Nishiura has shown that every steady state to the shadow system

$$u_t = D_u \Delta u + f(u,\xi) \quad \text{in} \quad \Omega \times \mathbb{R}_+, \qquad \tau\xi_t = \frac{1}{|\Omega|}\int_\Omega g(u,\xi)dx \quad \text{in} \quad \mathbb{R}_+, \tag{SS}$$

$$\partial_\nu u = 0 \quad \text{on} \quad \partial\Omega \times \mathbb{R}_+.$$

in a finite interval ($\Omega = [0,1]$) with certain conditions on f and g is unstable when u is neither constant nor monotone. Hence, if an inhomogeneous steady state (u,ξ) is stable, then u should be monotone. This result was generalized by Ni-Poláčik-Yanagida.

	Scalar equation	Shadow system
Interval	Chafee	Nishiura (Ni-Poláčik-Yanagida)
N-dimensional domain ($N \geq 2$)	Casten-Holland, Matano	?

We study the shape of the stable steady states of (SS) in a high-dimensional domain. It is well-known that there is a stable inhomogeneous steady state. Thus we have to answer the question: How do we describe the function defined in a high-dimensional domain? In this talk we try to describe the function, using the number and the locations of the critical points. The main result of this talk is about the shape of the stable patterns of (SS) in a disk.

Theorem A ([Mi06, Mi07]) *Let $D \subset \mathbb{R}^2$ be a disk, and let (u,ξ) be an inhomogeneous steady state to (SS) ($\Omega = D$), where*

$$\begin{gathered} f(\,\cdot\,,\,\cdot\,),\ g(\,\cdot\,,\,\cdot\,) \text{ are of class } C^2,\ f_\xi < 0,\ g_\xi < 0, \text{ and} \\ \text{there is a function } k(\xi) \in C^0 \text{ such that } g_u(u,\xi) = k(\xi) f_\xi(u,\xi). \end{gathered} \tag{2}$$

If (u,ξ) is stable for some $\tau > 0$, then u has exactly two critical points on $\overline{D}$, and those are on ∂D. See Figure 1. Hence if u has a critical point inside D, then (u,ξ) is unstable. In particular, the stable pattern does not have an interior peak.

This class of shadow system (2) includes the shadow system with the FitzHugh-Nagumo type nonlinearity and a special case of the Gierer-Meinhardt system ($p = r - 1$).

In this talk, we mention that the stable pattern for the shadow system is deeply related to the nonlinear "hot spots" conjecture of E. Yanagida.

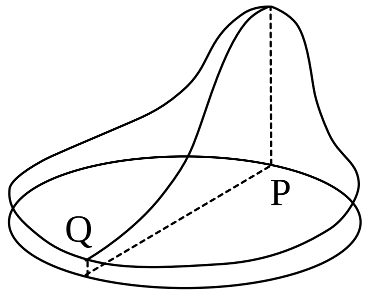

Figure 1: The shape of the stable pattens

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Decay and eventual local positivity for biharmonic parabolic equations

Filippo Gazzola
Politecnico di Milano, Italy

Abstract. We study existence and positivity properties for solutions to Cauchy problems for both linear and semilinear parabolic equations with the biharmonic operator as elliptic principal part. The self-similar kernel of the parabolic operator $\partial_t + \Delta^2$ is a sign changing function and the solution of the evolution problem with a positive initial datum may display almost instantaneous change of sign. We determine conditions on the initial datum for which the corresponding solution exhibits some kind of positivity behavior. We prove eventual local positivity properties both in the linear and semilinear case. At the same time, we show that negativity of the solution may occur also for arbitrarily large given time, provided the initial datum is suitably constructed.

Hot spots for the heat equation with a rapidly decaying negative potential

Kazuhiro Ishige
Tohoku University, Japan

Abstract. We consider the solution u of the Cauchy problem

$$(*) \qquad \begin{cases} \partial_t u = \Delta u - V(|x|)u & \text{in} \quad \mathbf{R}^N \times (0,\infty), \\ u(x,0) = \phi(x) & \text{in} \quad \mathbf{R}^N, \end{cases}$$

where $N \geq 2$, $\phi \in L^2(\mathbf{R}^N, e^{|x|^2/4}dx)$, and $V = V(r)(\not\equiv 0)$ is a nonnegative function behaving like

$$V(r) = O(r^{-\kappa}) \quad \text{as} \quad r \to \infty,$$

for some $\kappa > 2$. Then, under suitable additional conditions on the initial data ϕ, for example, $\phi \geq 0$ in $\mathbf{R}^N$, we see that

$$H(t) \equiv \left\{ x \in \mathbf{R}^N \, : \, u(x,t) = \max_{y \in \mathbf{R}^N} u(y,t) \right\} \neq \emptyset, \qquad t > 0.$$

We call a point $x \in H(t)$ a hot spot of the solution u at the time t. In this talk we study the large time behavior of the hot spots of the solution u. In particular, for our case, the hot spots tend to the space infinity as $t \to \infty$, and the main purpose of this talk is to study the following three subjects

(a) the rate for hot spots to tend to the space infinity as $t \to \infty$,

(b) the direction for hot spots to tend to the space infinity as $t \to \infty$,

(c) the number of hot spots for any sufficiently large t.

(This is a joint work with Yoshitsugu Kabeya.)

On the motion of polygonal curves by crystalline curvature flow with bulk effect

Tetsuya Ishiwata
Shibaura Institute of Technology, Japan

Abstract. We consider a motion of polygonal curves in the plane which is governed by crystalline curvature flow with the bulk effect:

$$\beta(N_j)V_j = U - H_j$$

and its generalization, where V_j, N_j and H_j denote an outward velocity, an outward normal vector and a crystalline curvature of the j-th facet(edge in 2 dimensional case) of solution curve, respectively. Here the function β describes an anisotropy of the mobility and U describes an effect of the bulk. This model equation is one of mathematical models of the interface motion of crystals(See [1, 3]). We discuss deformation behaviors of solution curves, especially, convexity phenomena, which means that the enclosed area of the solution curve becomes convex in finite time from non-convex initial curves. This phenomena is well-known for the motions of smooth curves by curve-shortening flow $v = -k$. For crystalline curvature flow without the bulk effect, there is partial results on this phenomena(See [4]) and there are no results for the case with the bulk effect. In this talk, we consider the expanding case, i.e. $U(t) > c > 0$ for some c, and show the convexity results under some conditions.

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Some recent estimates for nonlinear Dirichlet eigenvalues

Barbara Brandolini
Università di Napoli "Federico II", Italy

Abstract. We present some results concerning a priori estimates of solutions to nonlinear partial differential equations by means of geometrical properties of the domain. In the first part of the talk we establish a reverse Faber-Krahn inequality for the eigenvalue of the Dirichlet problem for the Monge Ampère operator. We show that, when computed on convex domains with fixed measure, it is maximal on ellipsoids. This result can be established by exploiting the affine invariant structure of such operator using either Blaschke-Santaló or Petty inequalities. In the second part we prove a Payne Weinberger type estimate for the first eigenvalue of the Dirichlet problem for the p-Laplacian operator. Roughly speaking, we provide an upper bound by means of the isoperimetric deficit of the domain.

Bifurcations in semilinear elliptic equations on thin domains

Toru Kan
Tohoku University, Japan

Abstract. We consider bifurcations of solutions of semilinear elliptic equations on thin domains in $\mathbb{R}^N$ ($N \geq 2$) whose width is not necessarily constant. If we regard thin domains as perturbation of a one-dimensional interval, some properties of solutions of equations on thin domains can be approximated by a reduced equation on the interval. The purpose of this talk is to show that solution branches for thin domains persist near those for the interval including bifurcation points. Proofs are obtained by applying the contraction mapping principle and the Lyapunov-Schmidt reduction.

Geometric inequalities related to a Bernoulli free boundary problem

Chiara Bianchini
Università degli studi di Firenze, Italy

Abstract. We study the non-linear interior Bernoulli problem: given a bounded regular domain Ω, we look for a function u and a domain K, contained in Ω, satisfying:

$$\begin{cases} \Delta_{\mathrm{p}} u = 0 & \text{in } \Omega \setminus K \\ u = 0 & \text{on} \quad \Omega \\ u = 1, |Du| = \tau & \text{on} \quad K, \end{cases}$$

for some given positive constant τ.

It is well known that this problem needs not have a solution for every value of τ. However in the convex case (that is when Ω is convex), there exists a positive constant Λ (named the Bernoulli constant), such that the problem has a solution if and only if $\tau \geq \Lambda$.

We prove that the Bernoulli constant satisfies a Brunn-Minkowski and an Isoperimetric type Inequality.
This work is a joint work with P. Salani

On the heat equation in a half-space with a nonlinear boundary condition

Tatsuki Kawakami
Tohoku University, Japan

Abstract. In this talk, we consider the heat equation with a nonlinear boundary condition,

$$\begin{cases} \partial_t u = \Delta u, & x \in \Omega, \ \ t > 0, \\ \partial_\nu u = u^p, & x \in \partial\Omega, \ \ t > 0, \\ u(x,0) = \phi(x), & x \in \Omega, \end{cases} \tag{1}$$

where $\Omega = \{x = (x', x_N) \in \mathbf{R}^N : x_N > 0\}$, $N \geq 1$, $\partial_t = \partial/\partial t$, $\partial_\nu = -\partial/\partial x_N$, $p > 1$, and $\lambda > 0$. Throughout this talk we assume that

$$\phi \in X \equiv \left\{ f \in L^\infty(\Omega) \cap L^2\left(\Omega, e^{|x|^2/4} dx\right) \ : \ f \geq 0 \text{ in } \Omega \right\}, \tag{2}$$

$$1 + 1/N < p, \quad (N-2)p < N. \tag{3}$$

The nonlinear boundary value problem (1) can be physically interpreted as a nonlinear radiation law, and has been studied by many mathematicians. In this talk we classify the large time behavior of positive solutions of (1) under the conditions (2) and (3). This is a joint work with Kazuhiro Ishige (Tohoku University).

Hardy-Rellich type inequalities with boundary terms and applications to semilinear biharmonic problems

Elvise Berchio
Dipartimento di Mathematica Politecnico di Milano, Italy

Abstract. We present a family of Hardy-Rellich type inequalities having boundary terms and where the optimal constants are not necessarily the classical Hardy-Rellich ones. When the domain is the unit ball many computations can be done explicitly and the exact values of the constants can be performed. We exploit this fact to investigate the so called " critical dimensions " for some semilinear biharmonic problems under Steklov boundary conditions.

Forward self-similar solution with a moving singularity for a semilinear parabolic equation

Shota Sato
Tohoku University, Japan

Abstract. We study the Cauchy problem for a semilinear parabolic equation. It was shown in our previous paper that in some parameter range, the problem has a time-local solution with prescribed moving singularities. Our concern in this talk is the existence of a time-global solution. By using a perturbed Haraux-Weissler equation, it is shown that there exists a forward self-similar solution with a moving singularity. This talk is a joint work with Eiji Yanagida (Tohoku University).

On Hardy inequalities with a remainder term

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Abstract. We study some improvements of the classical Hardy inequality . This inequality asserts that (see [6])

$$\int_{\Omega} |\nabla u|^2 \, dx \geq \frac{(N-2)^2}{4} \int_{\Omega} \frac{u^2}{|x|^2} dx, \quad \forall u \in H_0^1(\Omega), \tag{4}$$

where Ω is a bounded open set of $\mathbb{R}^N$ containing the origin, $N > 2$. The constant in (4) is the best possible; however it is not attained. Hence, a natural question is about to get an improvement of (4) by adding a suitable remaining term involving some norm of u or of the gradient of u. The first result in this direction was obtained by Brezis and Vazquez in [3] and subsequently many authors worked on this problem (see, for example, [2], [4], [5], [7]). In [1] we add to the right hand side of the inequality a term that depends on some Lorentz norms of u or of its gradient and we find the best values of the constants for remaining terms . In both cases we show that the problem of finding the optimal value of the constant can be reduced to a spherically symmetric situation. This result is new when the right hand side is a Lorentz norm of the gradient.

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A Liouville-type theorem for some Weingarten hypersurfaces

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Abstract. Let $F = F(s)$ be a C^2 symmetric function on Ω given by

$$\Omega = \{ \ s = (s_1, \cdots, s_N) \in \mathbb{R}^N \ : \ s_j > 0 \ \text{ for all } j = 1, \cdots, N \ \},$$

where $N \geq 2$. Assume that F satisfies

$$F_{s_j} > 0 \ \text{ for all } j = 1, \cdots, N \text{ and } \ 0 \geq [F_{s_i s_j}] \geq -2 \text{ diag} \left[\frac{1}{s_1} F_{s_1}, \cdots, \frac{1}{s_N} F_{s_N} \right] \ \text{ in } \Omega.$$

Consider an entire graph G in $\mathbb{R}^{N+1}$ defined by $x_{N+1} = u(x_1, \cdots, x_N)$ for a C^2 function $u = u(x)$ on $\mathbb{R}^N$, where $x = (x_1, \cdots .x_N) \in \mathbb{R}^N$. Let $\kappa_1(x), \cdots, \kappa_N(x)$ be the principal curvatures of G with respect to the upward normal vector to G at $(x, u(x))$ for $x \in \mathbb{R}^N$. Our main theorem is the following.

Theorem 1 *Suppose that there exist three real constants $R > 0, L > 0$ and c satisfying*

$$F(1 - R\kappa_1, \cdots, 1 - R\kappa_N) = c, \ |\nabla u| \leq L \ \textit{and} \ \max_{1 \leq j \leq N} \kappa_j < \frac{1}{R} \quad \textit{in} \ \mathbb{R}^N.$$

Then $c = F(1, \cdots, 1)$ and u is an affine function, that is, G is a hyperplane.

The case where $F = \sum_{j=1}^{N} \log s_j$ is related to the result of [MS]. We use the method of [MS] and the strong comparison principle of Giga and Ohnuma [GO].

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A variational characterization of the first eigenvalue of Δ_∞

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Abstract. We study the eigenvalue problem for the Δ_∞-operator with respect to existence and uniqueness. The approach is variational: first we prove existence for the first eigenfunction via $\Gamma-$convergence, then we study uniqueness in the subclass of "local minimizer".

Stabilization to equilibria in a supercritical semilinear heat equation

Eiji Yanagida
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Abstract. For the Fujita-type equation with a supercritical exponent in the sense of Joseph and Lundgren, it is known that there exists an ordered family of equilibria, and each equilibrium is asymptotically stable in an appropriate weighted space. In this talk, I discuss fundamental properties of convergent solutions, in particular, the relation between initial values and the behavior of solutions.

On complex-valued 2D eikonals: continuation past a caustic

Giorgio Talenti
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Abstract. Theories of monochromatic high-frequency electromagnetic fields have been designed by Felsen, Kravtsov, Ludwig and others with a view to portraying features that are ignored by geometrical optics. These theories have recourse to eikonals that encode information on both phase and amplitude – in other words, are complex-valued. The following mathematical principle is ultimately behind the scenes: any geometric optical eikonal, which conventional rays engender in some light region, can be consistently continued in the shadow region beyond the relevant caustic, provided an alternative eikonal, endowed with a non-zero imaginary part, comes on stage. The present talk is devoted to exploring such a principle in dimension 2. Investigated are a partial differential system that governs the real and the imaginary parts of complex-valued two-dimensional eikonals, and an initial value problem germane to it. In physical terms, such a problem amounts to detecting waves that rise beside, but on the dark side of, a given caustic. In mathematical terms, the problem shows two main peculiarities: on the one hand, degeneracy near the initial curve; on the other hand, ill-posedness in the sense of Hadamard. A number of technical devices come into play: hodograph transforms, artificial viscosity, and a suitable discretization. Approximate differentiation and a parody of the quasi-reversibility method are also involved. An algorithm is offered that restrains instability and produces effective approximate solutions.

Existence of maximizing functions for functionals of critical growth

Michinori Ishiwata
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Abstract. It is known that the classical Trudinger-Moser inequality (in bounded domains) has $\mathbb{R}^2$-versions. One of such extensions is observed by D. M. Cao. The associated variational problem is the maximizing problem for $I_\alpha(u) := \int_{\mathbb{R}^2}(e^{\alpha u^2} - 1)dx$ subject to $\|u\|_{H^1(\mathbb{R}^2)} = 1$. The existence of a maximizing function for this variational problem with critical exponent $\alpha = 4\pi$ is discussed by B. Ruf in detail. In this talk, we discuss the existence of a maximizer for this variational problem with $\alpha \leq 4\pi$ and try to reveal the difference between the problem defined on bounded domains and on $\mathbb{R}^2$.

Sharp estimates for linear Neumann eigenvalues and eigenfunctions

Cristina Trombetti
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Abstract. We prove a reverse Hölder inequality for the first nontrivial eigenfunction of the Neumann Laplacian. From this we deduce a lower bound for the first nontrivial eigenvalue in terms of the isoperimetric constant relative to the domain. We finally show that our results are aymptotically sharp at least in dimension two.

A local mountain pass type result for a system of nonlinear Schrödinger equations

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Abstract. We consider a singular perturbation problem for the following system of nonlinear Schrödinger equations:

$$\begin{aligned} -\varepsilon^2\Delta u_1 + V_1(x)u_2 &= \mu_1 u_1^3 + \beta u_1 u_2^2 \qquad \text{in } \mathbb{R}^N, \\ -\varepsilon^2\Delta u_2 + V_2(x)u_2 &= \mu_2 u_2^3 + \beta u_1^2 u_2 \qquad \text{in } \mathbb{R}^N, \\ u_1(x) > 0,\ u_2(x) &> 0 \quad \text{in } \mathbb{R}^N, \\ u_1(x) \to 0,\ u_2(x) &\to 0 \quad \text{as } |x| \to \infty. \end{aligned}$$

Here $N = 2$, 3 and μ_1, μ_2, β are positive constants satisfying $\mu_1\mu_2 - \beta^2 > 0$ and $V_1(x)$, $V_2(x) \in C^1(\mathbb{R}^N)$ are positive bounded functions.

For fixed $y \in \mathbb{R}^N$ we denote by $m(y)$ the least energy level for *vector solutions* of *freezed* equations:

$$\begin{aligned} -\Delta u_1 + V_1(y)u_2 &= \mu_1 u_1^3 + \beta u_1 u_2^2 \qquad \text{in } \mathbb{R}^N, \\ -\Delta u_2 + V_2(y)u_2 &= \mu_2 u_2^3 + \beta u_1^2 u_2 \qquad \text{in } \mathbb{R}^N. \end{aligned}$$

We assume that there exists a bounded set $\Lambda \subset \mathbb{R}^N$ such that

$$\inf_{x\in\Lambda} m(x) < \inf_{x\in\partial\Lambda} m(x)$$

and we will find a family of positive solutions which concentrates to a point in Λ as $\varepsilon \to 0$.

This is a joint work with N. Ikoma.

Minimal surfaces in CR manifolds

Andrea Malchiodi
SISSA Trieste, Italy

Abstract. We define a notion of area and mean curvature for hypersurfaces embedded in CR manifolds, modeled on the Heisenberg group. We discuss existence and regularity issues for the corresponding minimal and constant meancurvature surfaces.

Radial symmetry of solutions for an overdetermined boundary-value problem in exterior domains

Andrea Colesanti
Università di Firenze, Italy

Abstract. The subject of the talk is the following overdetermined problem:

$$\begin{cases} \Delta u = 0 \quad \text{in } \mathbf{R}^n \setminus \Omega \\ u = \text{const.} \quad \text{on } \partial\Omega \\ \|\nabla u\| = \text{const.} > 0 \quad \mathcal{H}^{n-1}\text{-a.e. on } \partial\Omega \end{cases}$$

where Ω is a bounded open subset in $\mathbf{R}^n$ with Lipschitz boundary and $\mathcal{H}^{n-1}$ is the $(n-1)$-dimensional Hausdorff measure. A general conjecture concerning this problem, attributed to Gruber, is that a solution may exist only if Ω is a ball and in this case u is radially symmetric. This conjecture has been proved in dimension $n = 2$, and, with additional assumptions on the domain Ω, in higher dimension. In particular, Reichel (1997) proved it under the assumption that $\partial\Omega$ is of class $C^{2,\alpha}$, while Mendez and Reichel (2000) established the result for Ω convex. In a recent paper, in collaboration with Reichel and Salani, we extended this result to the class of sets with positive reach, introduced by Federer (1959), which encloses domains with $C^{2,\alpha}$ boundary and convex domains.

Asymptotic behavior of solutions to nonlinear elliptic eigenvalue problems

Tetsutaro Shibata
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Abstract. We consider the nonlinear eigenvalue problems

$$-\Delta u = f(\lambda, u, \nabla u) \text{ in } \Omega, \tag{1}$$
$$u > 0, \text{ in } \Omega, \tag{2}$$
$$u = 0 \text{ on } \partial\Omega, \tag{3}$$

where $\Omega \subset \mathbf{R}^N$ is a smooth bounded domain and $\lambda > 0$ is a parameter. The typical examples of $f(\lambda, \xi, \eta)$ are as follows.

$$\begin{aligned} f(\lambda, \xi, \eta) &= \lambda \sin \xi, \\ f(\lambda, \xi, \eta) &= \lambda \sin \xi - g(\xi), \\ f(\lambda, \xi, \eta) &= \lambda \xi - \xi^p - |\eta|^m \quad (1 < p, \;\; m = 0 \;\; or \;\; 1 \le m < 2, \;\; N = 1). \end{aligned}$$

Further, the prototype of $g(\xi)$ is $g(\xi) = |\xi|^{p-1}\xi \;\; (p > 1)$.

One of the most interesting problems to study in these problems is to clarify the asymptotic shapes of the solutions u_λ as $\lambda \to \infty$. We know that the asymptotic shape of these solutions are almost flat inside Ω and develop boundary layers when $\lambda \gg 1$. The purpose of this talk is to study precisely the asymptotic behavior of solutions u_λ as $\lambda \to \infty$. Both PDE and ODE cases will be discussed.

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Critical points of solutions of quasi-linear degenerate elliptic equations in the plane

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Abstract. We show that the minimizer of a certain strictly convex **non-differentiable** functional satisfy a quasi-linear degenerate elliptic equation in the viscosity sense. We then prove that in the plane, due to such a lack of differentiability, the critical points of the minimizer **cannot** be isolated. We also discuss how some sort of **stream function** can be associated to the minimizer.