A torsion problem with non-constant boundary data

Chiara Bianchini (Università di Firenze)

We investigate an overdetermined torsion problem, with a non-constant positively homogeneous boundary constraint on the gradient. We interpret this problem as the Euler equation of a shape optimization problem, we prove existence and regularity of a solution. Moreover several geometric properties of the solution are shown.

This talk is based on a joint work with A. Henrot (Nancy) and P. Salani (Firenze).

Solutions of elliptic equations with a level surface parallel to the boundary: stability of the radial configuration

Giulio Ciraolo (Università di Palermo)

Positive solutions of homogeneous Dirichlet boundary value problems for certain elliptic equations must be radially symmetric if just one of their level surfaces is parallel to the boundary of the domain. In this talk, for the elliptic case, we show the stability counterpart of that result. In fact, we show that if the solution is almost constant on a surface at a fixed distance from the boundary, then the domain is almost radially symmetric, in the sense that its boundary is contained in a circular annulus of radii r_e and r_i, where the difference $r_e - r_i$ linearly controlled by a suitable norm of the deviation of the solution from a constant. The proof relies on a quantitative study of the method of moving planes.

This talk is based on joint works with R. Magnanini and V. Vespri (Firenze) and S. Sakaguchi (Tohoku).

Stability for parabolic quasiminimizers

Yohei Fujishima (Osaka University)

We study parabolic quasiminimizers, which satisfies parabolic variational inequalities. In this talk, we show that, under a suitable regularity condition on the boundary, parabolic Q-quasiminimizers related to the parabolic p-Laplace equations with given boundary data are stable with respect to parameters Q and p.

Stability results for Urysohn's inequalities involving elliptic operators

Daria Ghilli (Università di Padova)

The aim of the talk is to introduce some recent results about Urysohn's inequalities for some functionals involving elliptic operator. In particular, we present a stability result for the Borell-Brascamp-Lieb inequality in terms of the Hausdorff distance between the support sets of the functions involved. Then we show how it is possible to apply the previous result in order to prove analogous stability results for Urysohn's inequalities involving elliptic operators, such as the Urysohn inequality for the torsional rigidity.

This talk is based on a joint work with P. Salani (Firenze).

Sharp constants of Hardy inequalities for a mean oscillation

Norisuke Ioku (Ehime University and Università di Firenze)

The aim of the talk is to present some results on sharp constants in Hary inequalities for a mean oscillation:

$$C_{N,p}\int_{\mathbb{R}^N}\frac{\left(\tilde{u^\sharp}(x)-u^\sharp(x)\right)^p}{|x|^p}dx\le\int_{\mathbb{R}^n}|\nabla u(x)|^p\,dx \tag{1}$$

for every $u\in W^{1,p}(\mathbb{R}^N)$, where $N\ge 2$, $1\le p<\infty$, $u^\sharp$ is a radially symmetric non-dicreasing rearrangement of u, and $\tilde{u^\sharp}(x)$ is a mean integral of $u^\sharp$ on $B_{|x|}(0)$. The inequality without its sharp constant essentially proved by Alvino-Trombetti-Lions [1] and Kolyada [3]. It is known that the inequality (1) implies Sobolev-Lorentz embeddings $W^{1,p}(\mathbb{R}^N)\subset L^{p^*,p}(\mathbb{R}^N)$ if $1\le p<N$, and the critical log-Hardy inequality if $p=N$.

In this talk, we consider sharp constants of (1) and related variational problems. This is a joint work with Professor Michinori Ishiwata (Fukushima University).

References

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Characterization of ellipsoids as K-dense sets

Michele Marini (Scuola Normale Superiore Pisa)

Let K be any convex body in the Euclidean space containing the origin. A measurable set G with finite and positive Lebesgue measure is said to be K-dense if, for any fixed positive r the measure of the intersection of G with $x+rK$ is constant when x varies on the boundary of G (here, $x+rK$ denotes a translation of a dilation of K). When K is the Euclidean ball B, these sets have been studied in connection with the time-invariant level surfaces of solutions of the heat equation and it has been shown that any B-dense set is a ball; we will show that in the general case the above property characterizes ellipsoids, more precisely we prove that if G is K-dense, then both G and K must be homotetic to the same ellipsoid.

This talk is based on a joint work with R. Magnanini (Firenze).

Regularity of solutions to the obstacle problem for the parabolic biharmonic equation

Shinya Okabe (Tohoku University)

We study the regularity of solutions to the obstacle problem for the parabolic biharmonic equation. We analyze the problem via an implicit discretization, and we prove some regularity properties of the solution. This talk is based on a joint work with M. Novaga (Università di Pisa).

Final state problem for a quadratic nonlinear Schrödinger system in two dimensions

Kota Uriya (Tohoku University)

We are concerned with the asymptotic behavior in time of solutions to a system of quadratic nonlinear Schrödinger equations in two dimensions. We say that the solution of nonlinear Schrödinger equation is asymptotically free if the solution behaves like a solution of the free Schrödinger equation as time tends to infinity. In general, quadratic nonlinearities in two dimensions are critical in the sense of asymptotic behavior of solutions, that is, it is not clear whether there exists an asymptotically free solution or not. It is known that the parameters which stand for the mass influence to the asymptotic behavior of solutions. Under the mass resonance condition, we show the existence of a solution to the system which is neither asymptotically free nor modified free solution.