

# Analysis on Graphs in Sendai 2011

Date: 2011 Feb. 21<sup>st</sup>.

Place: Graduate School of Science, Tohoku University, 理学合同棟 508 B・C

## Program

- 10:00–10:50** Tomoyuki Shirai (Kyushu University)  
Random analytic functions and their zeros
- 11:00–11:50** Akihiro Munemasa (Tohoku University)  
Accumulation points of the smallest eigenvalues of graphs
- 14:00–14:50** Nobuaki Obata (Tohoku University)  
Asymptotic Spectral Analysis of Large Graphs  
~~ A Quantum Probabilistic Approach
- 15:00–15:50** Fumihiko Nakano (Gakushuin University)  
Domino tilings with impurities
- 16:00–16:50** Pavel Exner (Czech Academy of Sciences)  
Vertex approximations in quantum graphs

共催・日本学術振興会 科学研究費補助金 基盤研究 (A) 20244002 (代表:小谷元子)

・CREST: 数学と諸分野の協働によるブレークスルーの探索

「離散幾何学から提案する新物質創成と物性発現の解明」 (代表:小谷元子)

## Abstract

### **Tomoyuki Shirai (Kyushu University)**

Title: Random analytic functions and their zeros

Abstract: After recalling the notions of Gaussian analytic functions and determinantal (or fermion) point processes, we discuss a central limit theorem for a certain class of random analytic functions including random power series with i.i.d. random coefficients. Also we discuss a limit theorem for the zeros of random analytic functions.

### **Akihiro Munemasa (Tohoku University)**

Title: Accumulation points of the smallest eigenvalues of graphs

Abstract: The method of Alan Hoffman developed in 1970's was formalized by Woo and Neumaier in 1995. In this talk, we proceed further to introduce decompositions of Hoffman graphs, and reprove Hoffman's theorem on the smallest eigenvalues of graphs in the new formalism.

### **Nobuaki Obata (Tohoku University)**

Title: Asymptotic Spectral Analysis of Large Graphs

~~ A Quantum Probabilistic Approach

Abstract: Quantum probability theory provides a framework of extending the measure-theoretical (Kolmogorovian) probability theory. The idea traces back to von Neumann (1932), who, aiming at the mathematical foundation for the statistical questions in quantum mechanics, initiated a parallel theory by making a selfadjoint operator and a trace play the roles of a random variable and a probability measure, respectively. During the last 25 years quantum probability theory has developed considerably with wide applications. Our theme is relatively new and further development seems promising.

In this talk I will review how quantum probabilistic ideas are applied to spectral graph theory, in particular, to the study of asymptotic spectral distributions of large graphs (or growing graphs). We have so far developed three tools: (i) quantum decomposition and interacting Fock spaces; (ii) various concepts of independence and associated central limit theorems; (iii) partition statistics and moment-cumulant formulae. The method of quantum decomposition has been proved to be effective when the graph structure is reduced in a sense to a one-sided (finite or infinite) path graph because the theory of orthogonal polynomials of one-variable is fully applicable. It is apparently important to weaken this restriction and to go further. I will illustrate the basic idea of quantum decomposition and report some recent attempts in this line.

**Fumihiko Nakano (Gakushuin University)**

Title: Domino tilings with impurities

Abstract: We consider the dimer problem on a planar non-bipartite graph, where there are two types of dimers one of which we regard as impurities. Computer simulations imply a tendency that impurities are attracted to the boundary, which is the motivation to study this particular graph. We show that (1) the local move connectedness yielding an ergodic Markov chain on all possible dimer coverings, and (2) a bound of the number of dimer coverings and that of the probability of finding an impurity at a given edge.

**Pavel Exner (Czech Academy of Sciences)**

Title: Vertex approximations in quantum graphs

Abstract: It is a longstanding problem how to understand the coupling in vertices of a quantum graph using approximations, either by a family of appropriate "fat graphs" or by operators on the graph itself. In particular, within an approximation by Neumann Laplacians on a tube network the squeezing limit yields only the free (or Kirchhoff) boundary conditions. In this talk I will report first a recent result coming from a common work with Olaf Post: it will be shown that adding families of suitably scaled potentials to those Laplacians one can get spectrally nontrivial vertex couplings, including those with wave functions discontinuous at the vertices. Furthermore, I will describe a fresh result obtained together with Taksu Cheon and Ondřej Turek on approximations by Schrödinger operators on graphs which shows a way how the problem can be solved in full generality.

