Lecture Series on Navier-Stokes Equations

Part I: From the Theory of Very Weak Solutions to Regularity of Weak Solutions

Part II: Incompressible Fluid Flow Past or Around Rotating Bodies

Reinhard Farwig
Technische Universität Darmstadt
farwig@mathematik.tu-darmstadt.de.

The first part of the lectures deals with the theory of *very weak solutions* to the stationary and instationary Stokes and Navier-Stokes equations in a three-dimensional domain. Very weak solutions define a new class of solutions with no differentiability properties and not necessarily of finite energy, but which are unique since they belong to *Serrin's class* $L^s(0,T;L^q(\Omega))$ where $\frac{2}{s}+\frac{3}{q}=1$, $s>2$, $q>3$. The aim of the lectures is to develop and to apply this theory to questions of regularity of weak solutions of the instationary Navier-Stokes equations in the sense of Leray-Hopf and to prove local or even global in time or space regularity beyond *Serrin's condition*.

In the second part of this lecture series we consider the flow of an incompressible viscous fluid around or past a rotating rigid body. After a coordinate transform we get a modified Navier-Stokes system with a first order linearly growing term which is *not* subordinate to the Laplacean and requires sophisticated tools from *Harmonic Analysis* to get L^q-estimates even for the linearized stationary whole space problem. Moreover, we discuss spectral properties of the modified Stokes/Oseen operator which generates a bounded, but definitely *not analytic* C^0-semigroup.